Introduction to mesh generation for simulation

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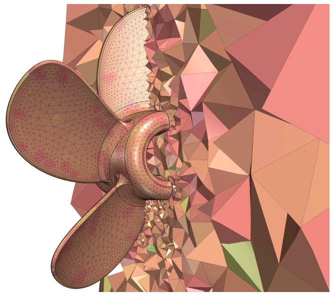
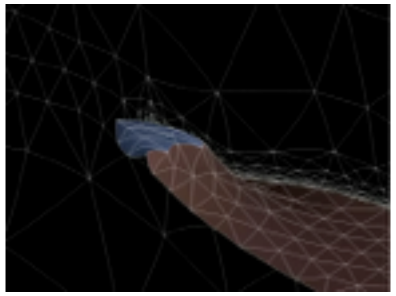
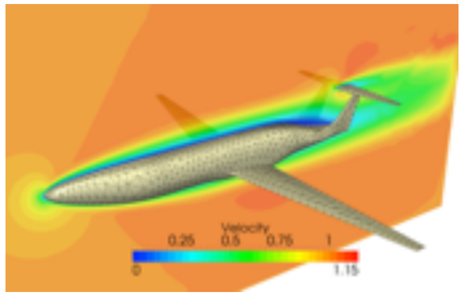
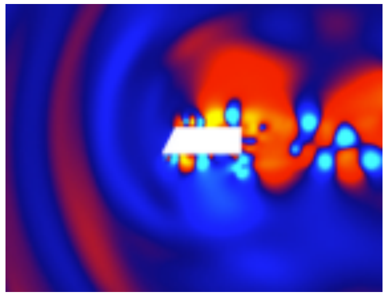
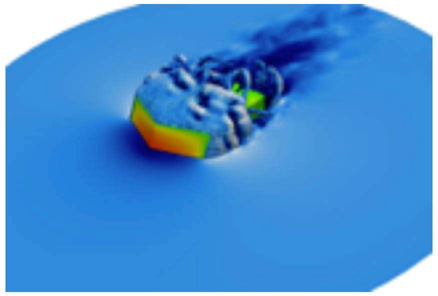
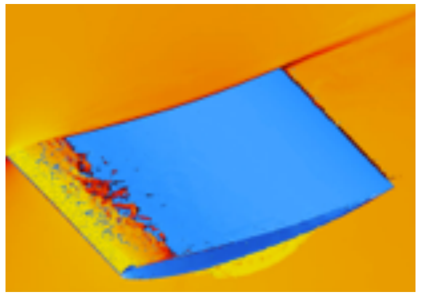
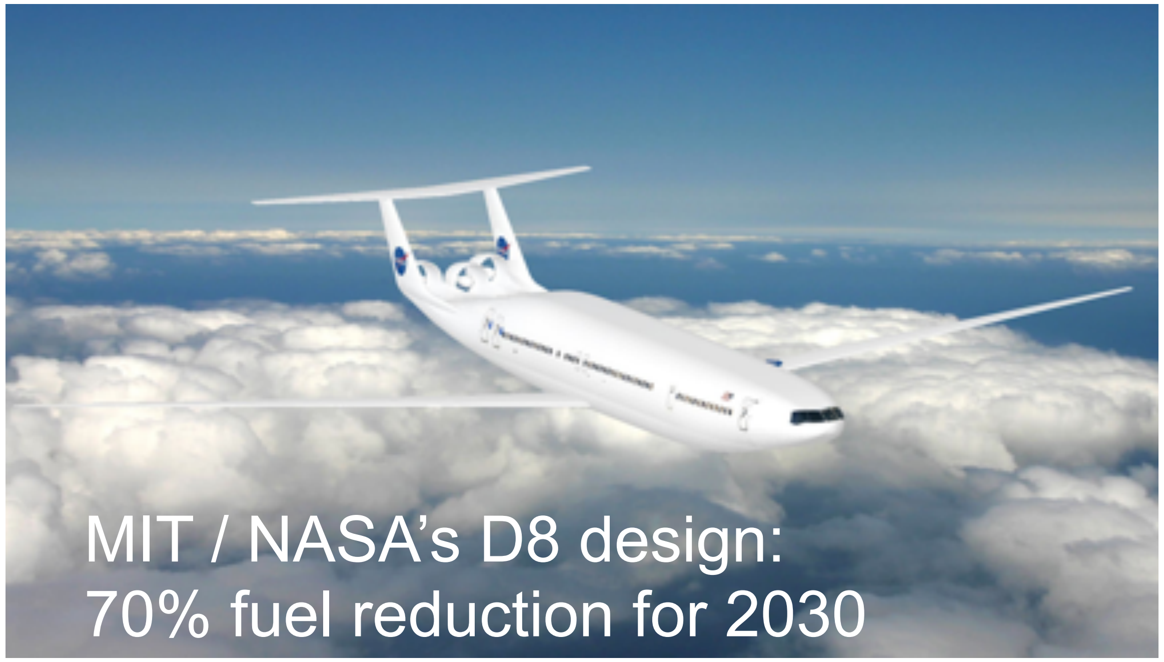
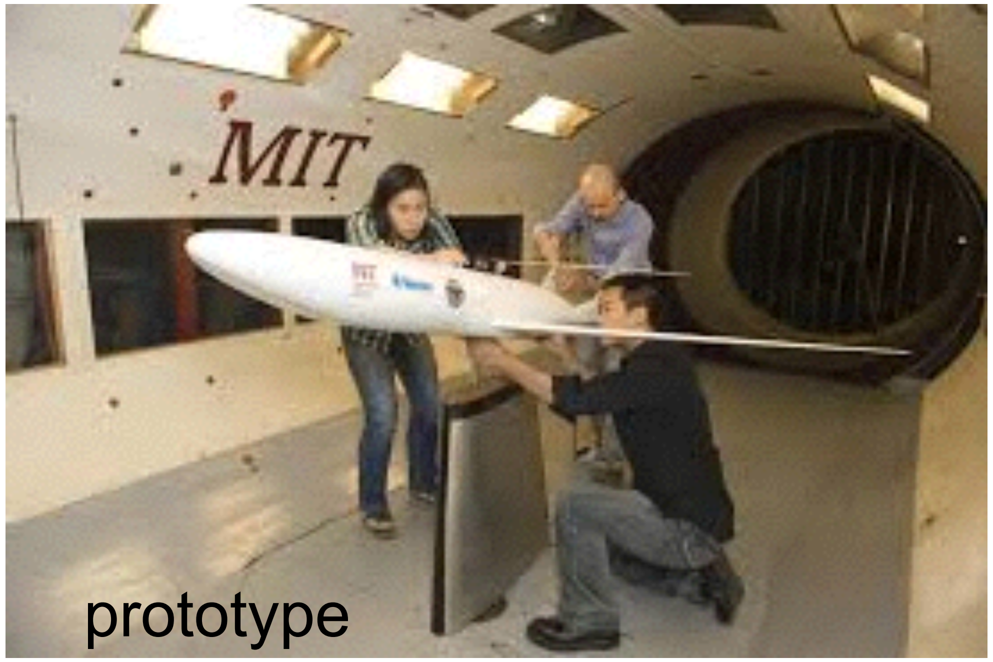







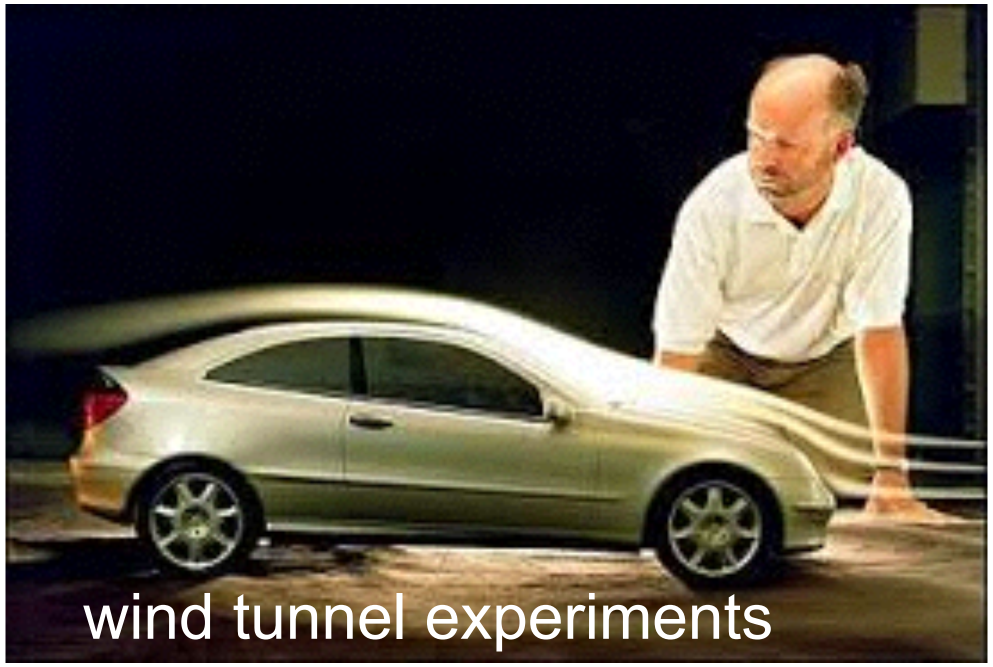
Motivation: high-fidelity flow simulation

- **Transportation:** reduce emissions and energy consumption

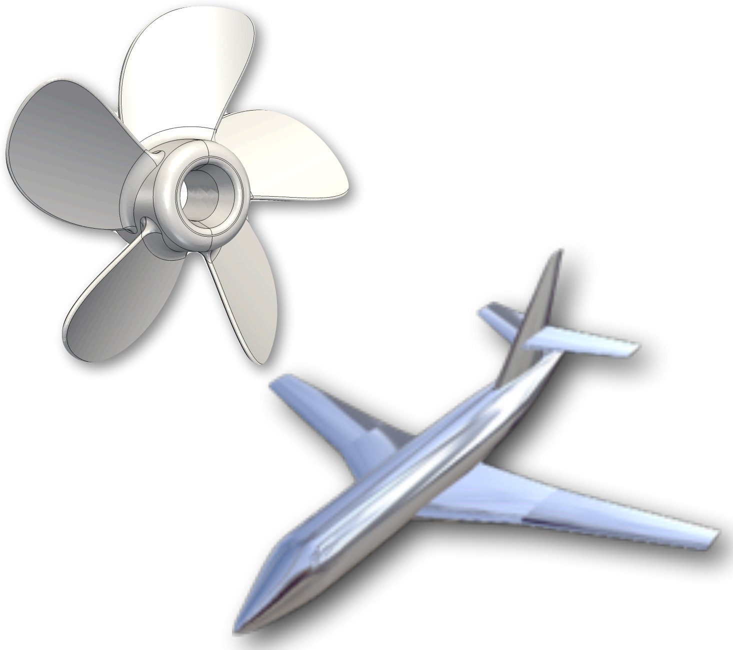


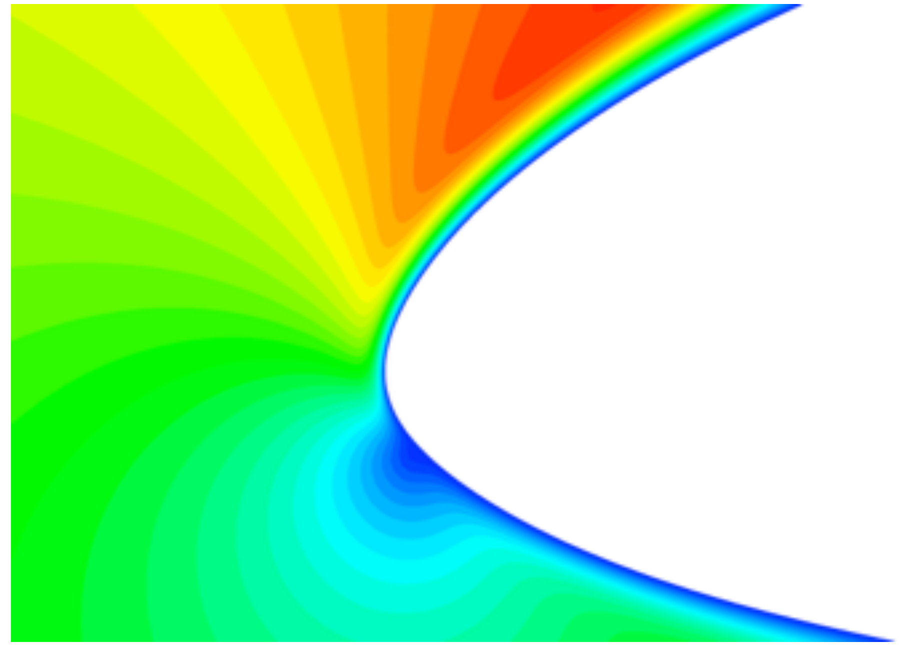


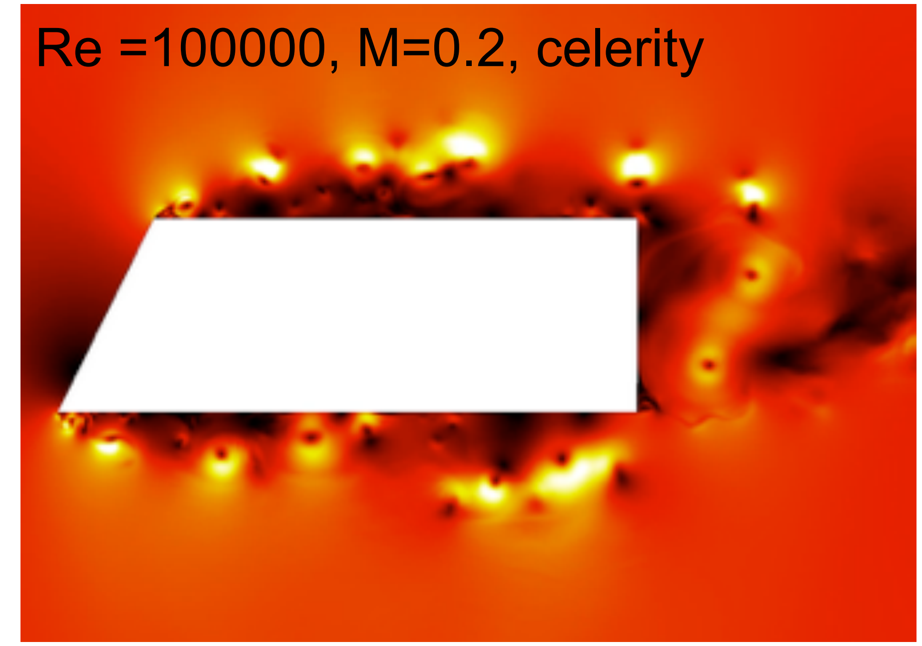
- **Exploring** automotive and aeronautical **designs** is **expensive**:
  - prototypes, experiments, wind tunnel, …
- **Accuracy** in aerodynamics is **extremely demanding**:
  - wind tunnel error in drag prediction: 1%
- **But**, simulations do not meet this goal!

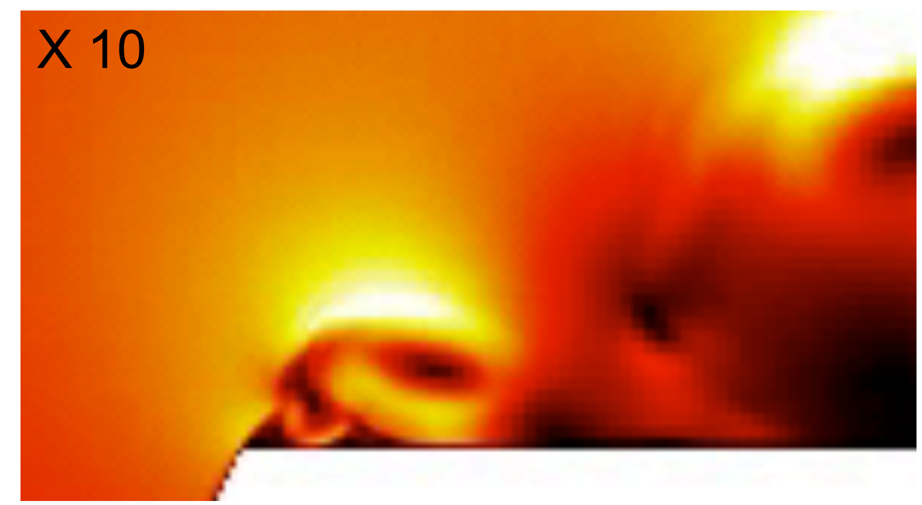


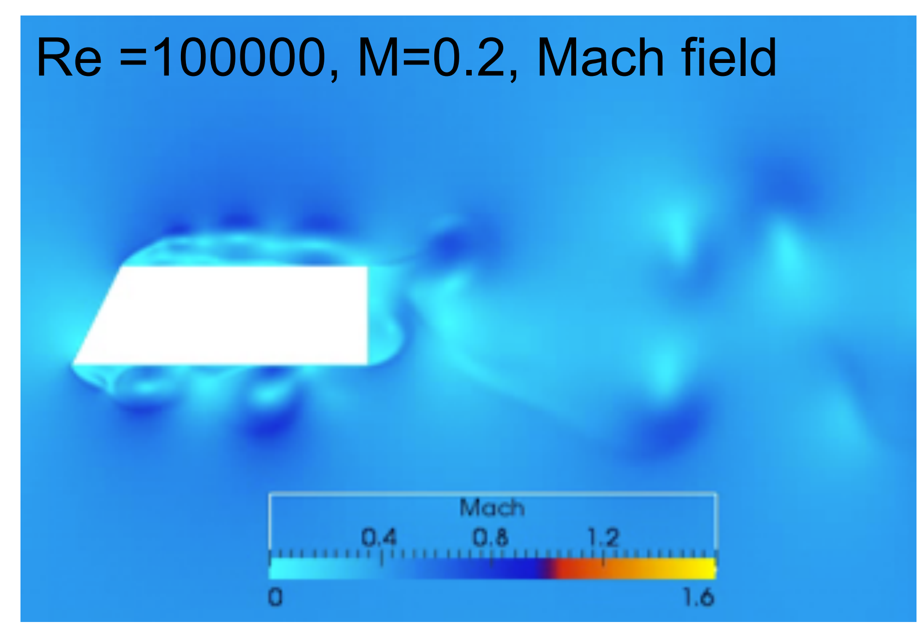
Challenges: high-fidelity flow simulation









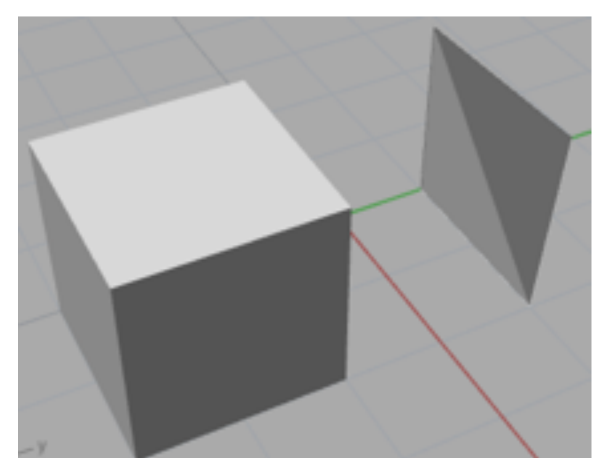


- **Geometrical**: accuracy, complexity, curved boundaries, sharp features …
- **Physical**: accurate model, boundary layers, separation, turbulence, …
- **Numerical**: accuracy, dissipation, dispersion, condition numbers, convergence, …
- **Computational**: rate of convergence, FLOPS, communication, parallelization, memory footprint, …

Computational methods for simulation

- *Partial differential equations* (PDEs) as a model for physics: Navier-Stokes, Euler, Darcy's law, Maxwell, …
- *Computational method*: from continuum PDEs to a a finite set of equations and degrees of freedom (DOFs)
  - The discrete solution approximates the continuum solution
  - The discrete problem can be solved with a finite number of operations
- The equations and DOFs are determined by a finite decomposition (*mesh*) of the space in *polytopes*: segments (1D), polygons (2D), polyhedra (3D), … referred as *elements*.

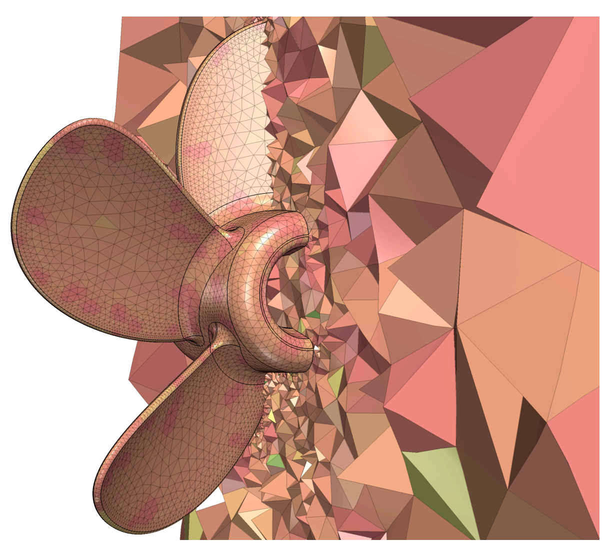
Polygons 2D: quadrilateral, triangle



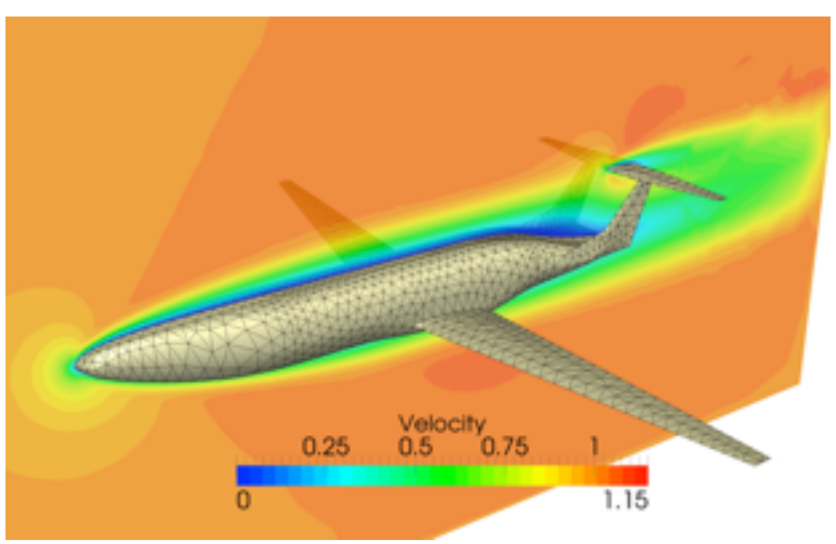
Polyhedra 3D: hexahedron, tetrahedron

Mesh generation

- *Mesh generation*: used in a daily basis by computational engineers to obtain predictions on complex geometries.
  - *Structured* methods require a regular mesh: finite differences
  - *Unstructured* methods allow irregular meshes: finite volumes, finite elements (continuous or discontinuous), …
- *Geometrical flexibility*: dealing with complex geometries
  - Structured methods are faster but have lower geometrical flexibility
  - Unstructured methods are slower but have higher geometrical flexibility



Mesh for a propeller



MIT / NASA's D8

Importance of meshing to overcome these challenges

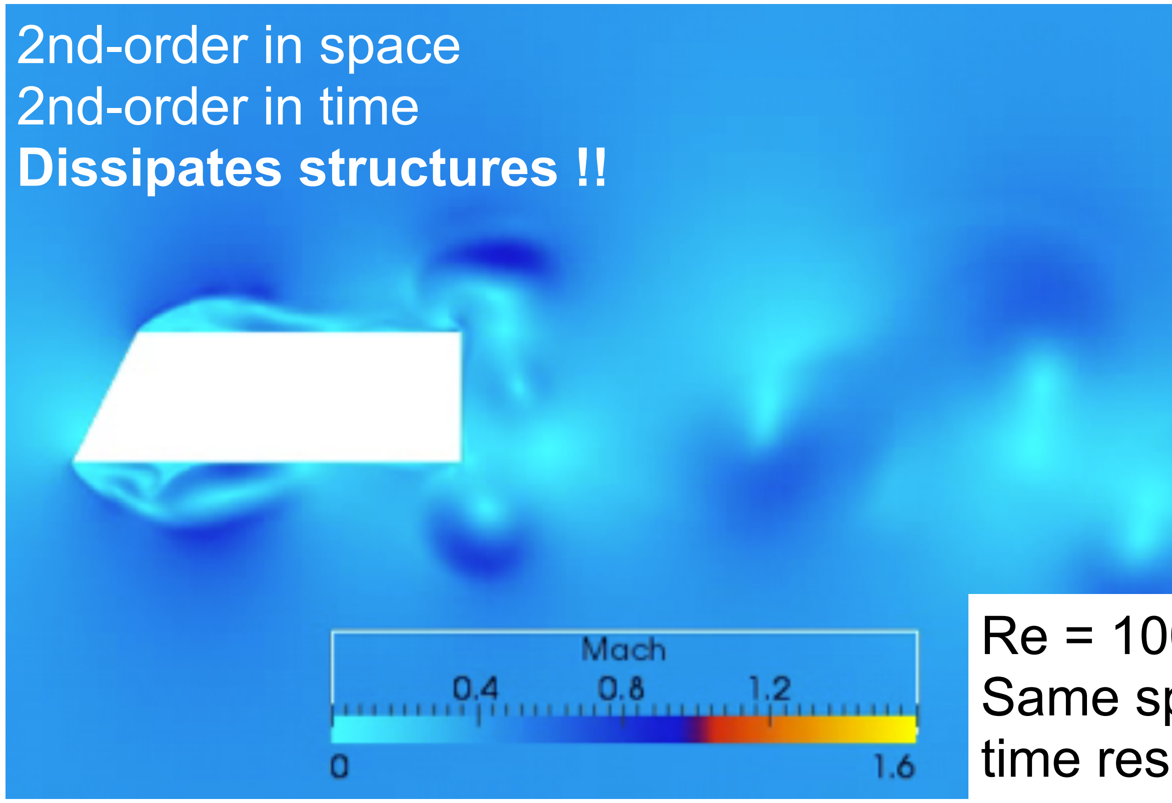
Discretizing (meshing) for high-fidelity simulation:

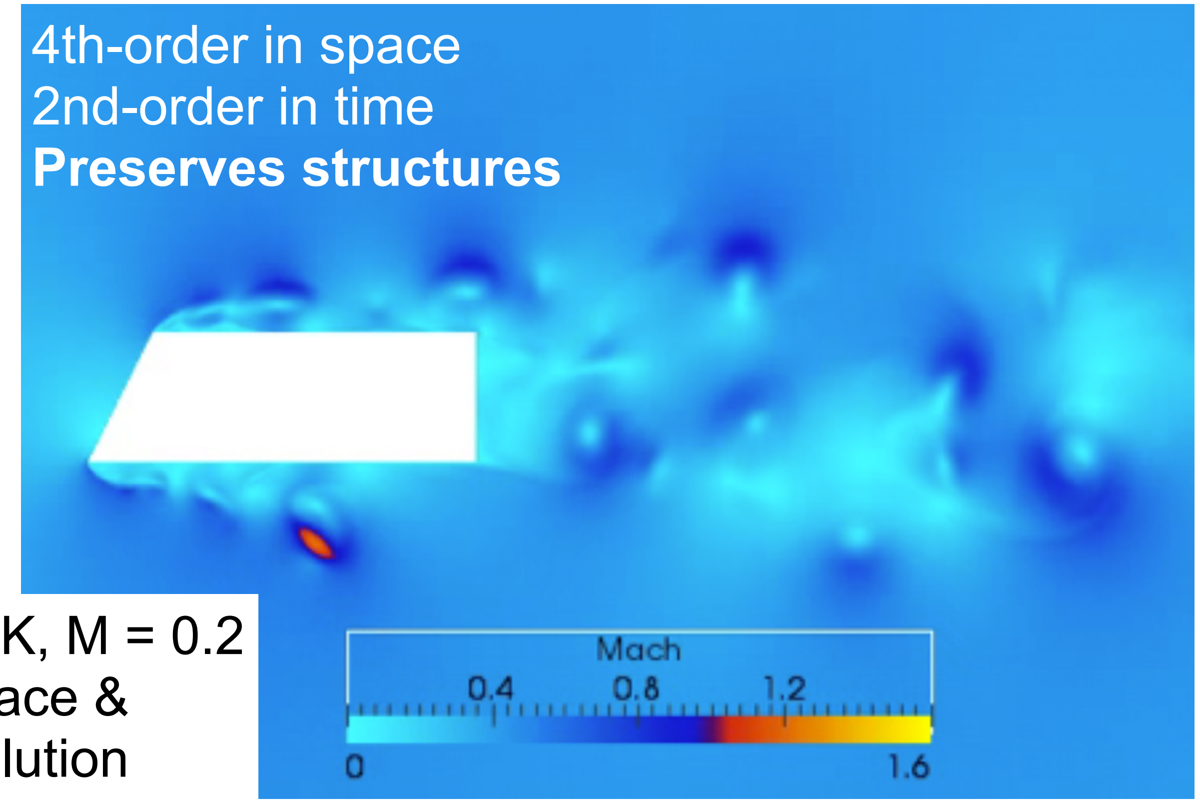
- geometrical fidelity
- physical fidelity
- numerical fidelity
- computational efficiency

High-order: low dissipation & dispersion

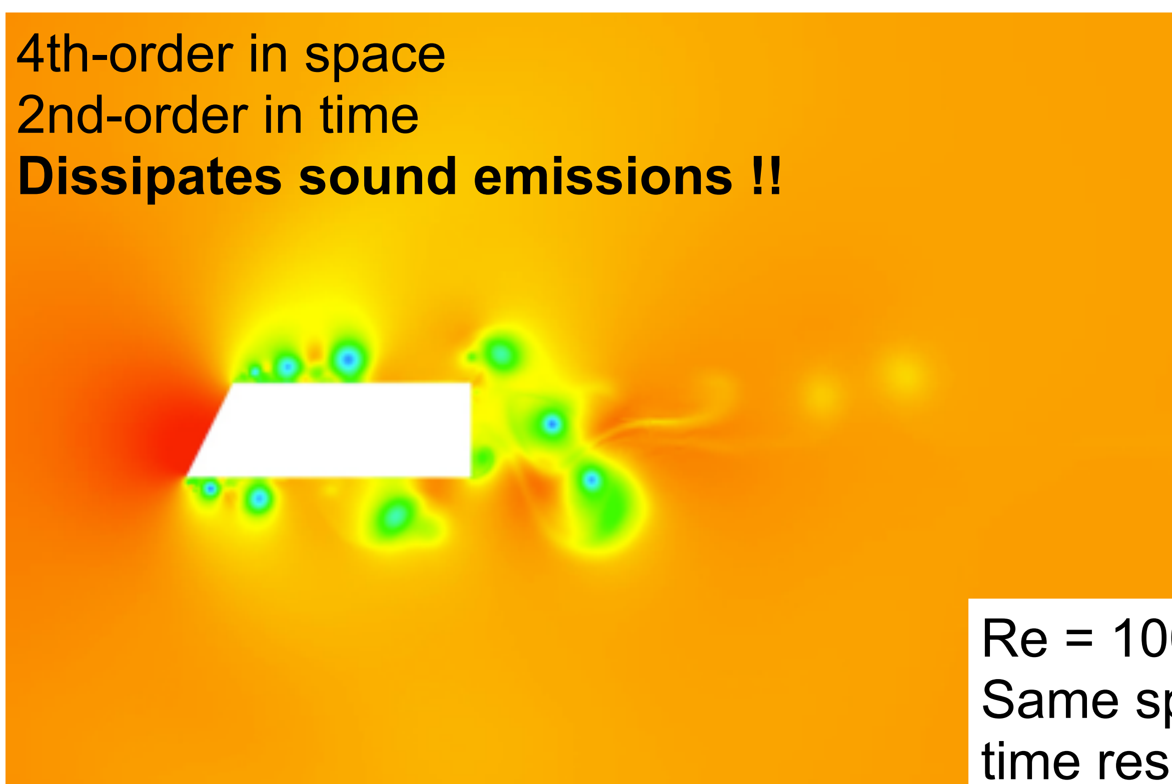
with N.C. Nguyen & J. Peraire

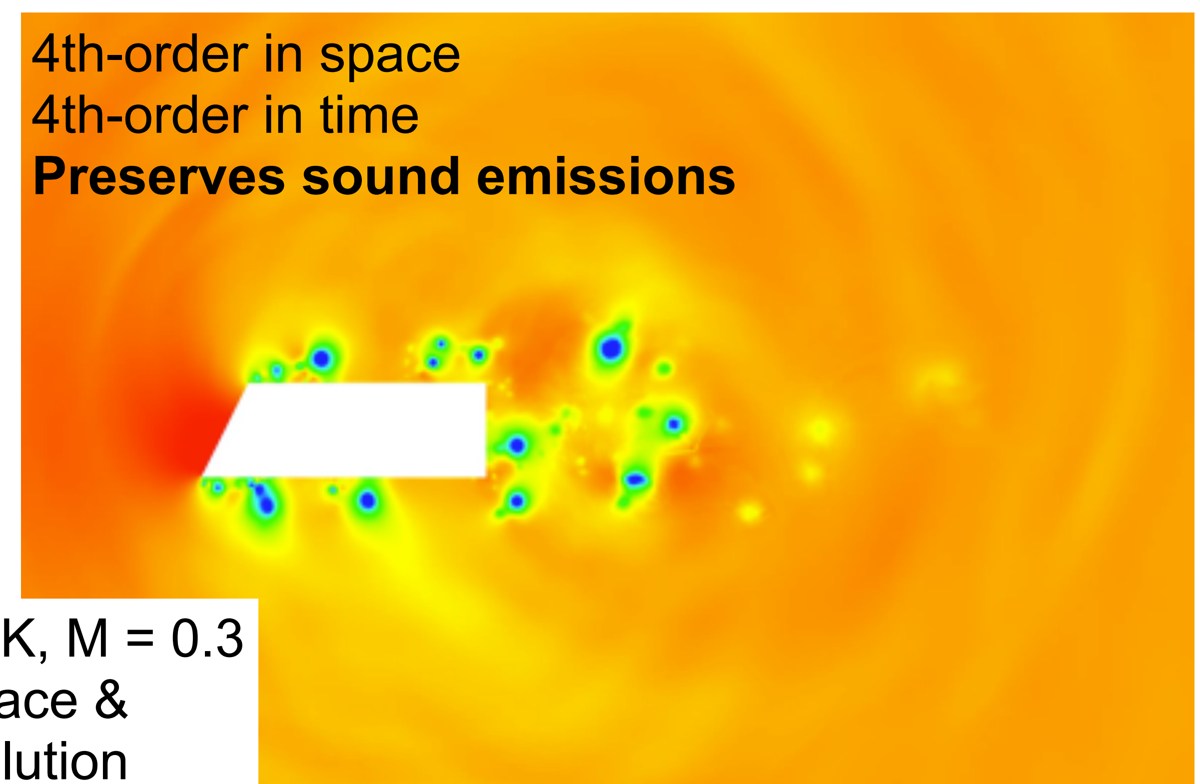
- Example: Compressible NS & Implicit Large Eddy Sim. (ILES) & high-order & HDG





Re = 100K, M = 0.2
Same space &
time resolution

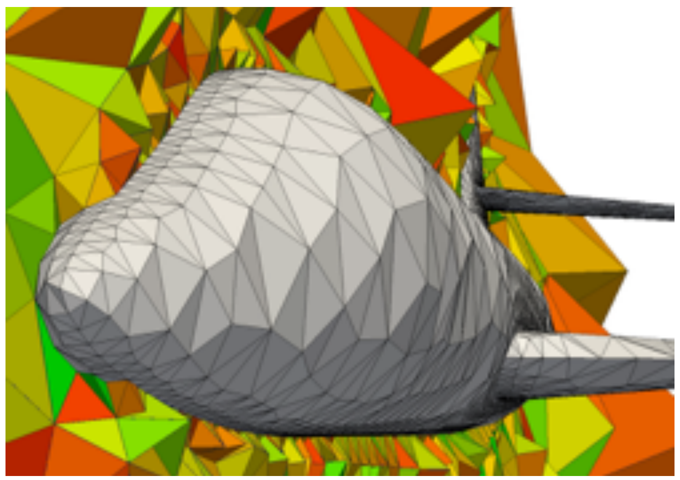




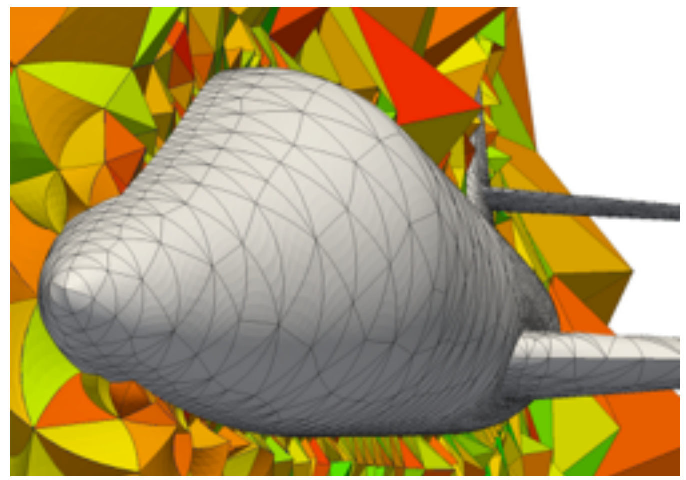
Re = 100K, M = 0.3
Same space &
time resolution

Curved boundaries & mesh quality are critical

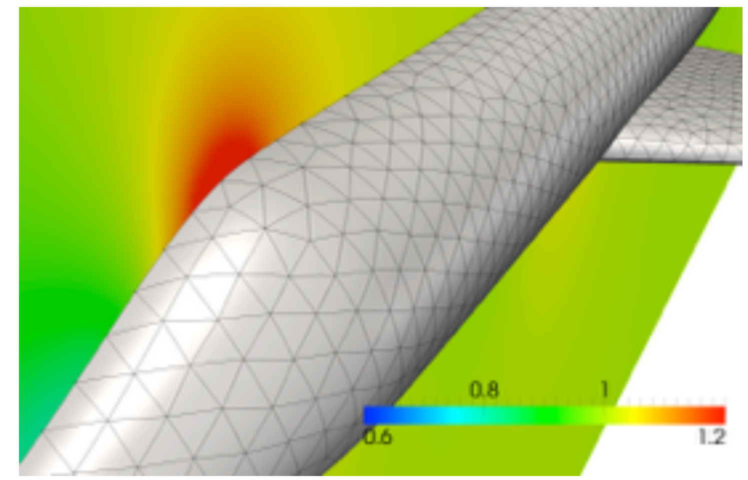
- 5th order approximation for inviscid flow: $\alpha = 0, M_\infty = 0.6, p = 4$
  - Straight-sided impedes convergence: artificial separation & entropy
    (as elucidated first for 2D cases by Bassi & Rebay'97)



Straight-sided: no convergence

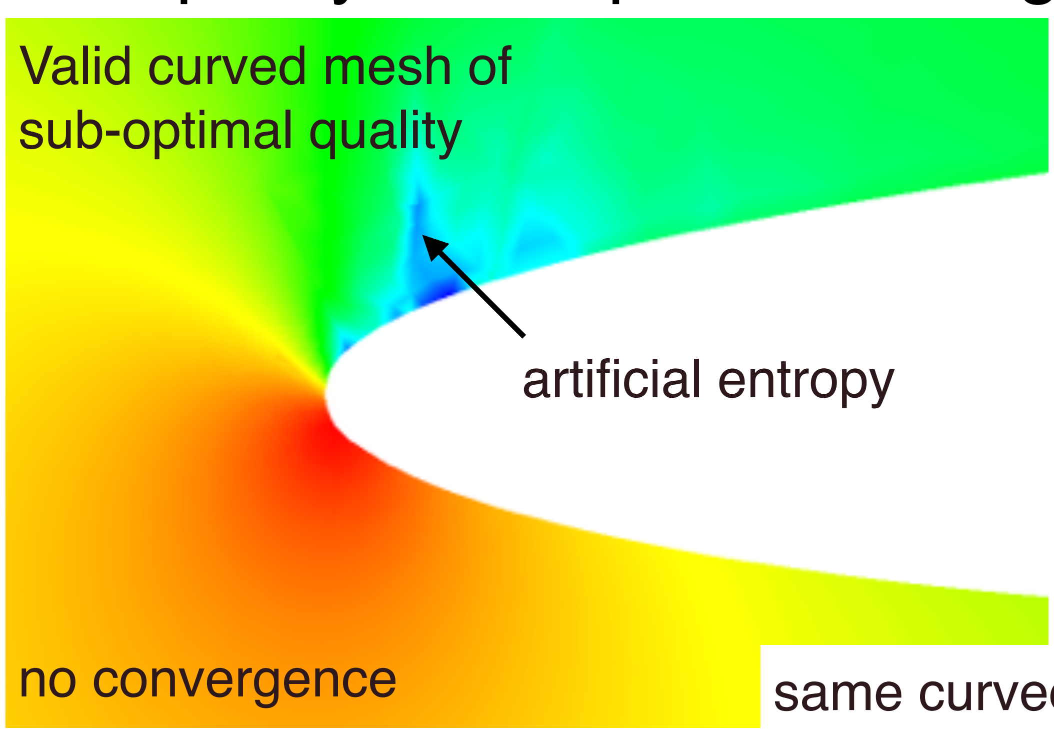


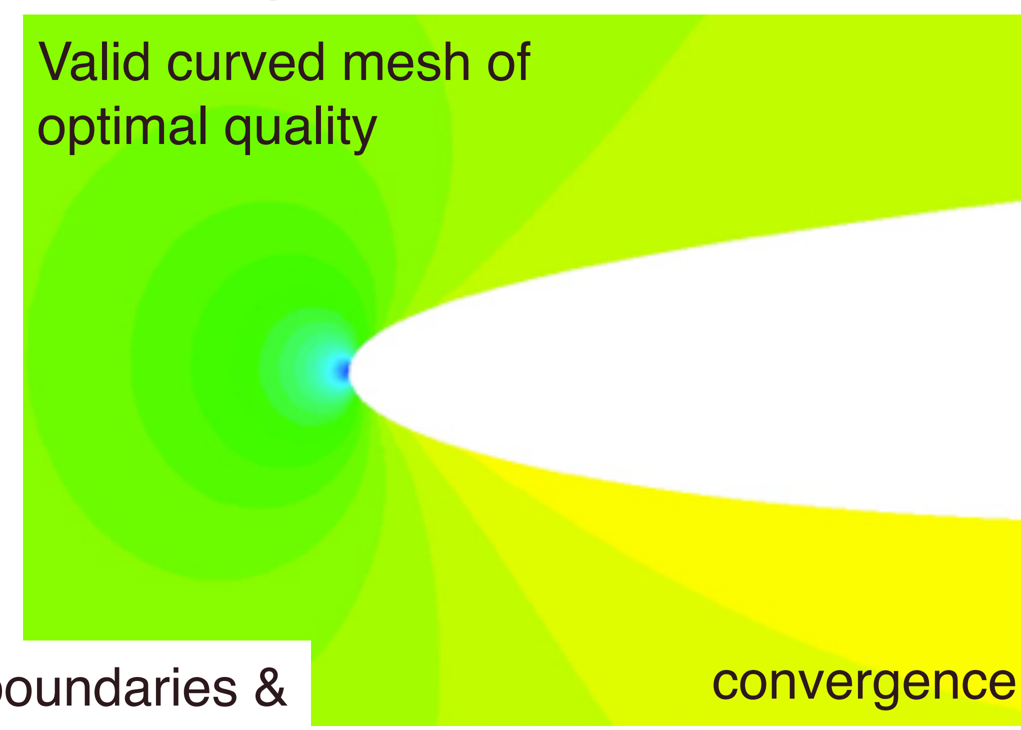
Curved: convergence



Curved: velocity magnitude

  - Low-quality can impede convergence: shape, smoothness, …

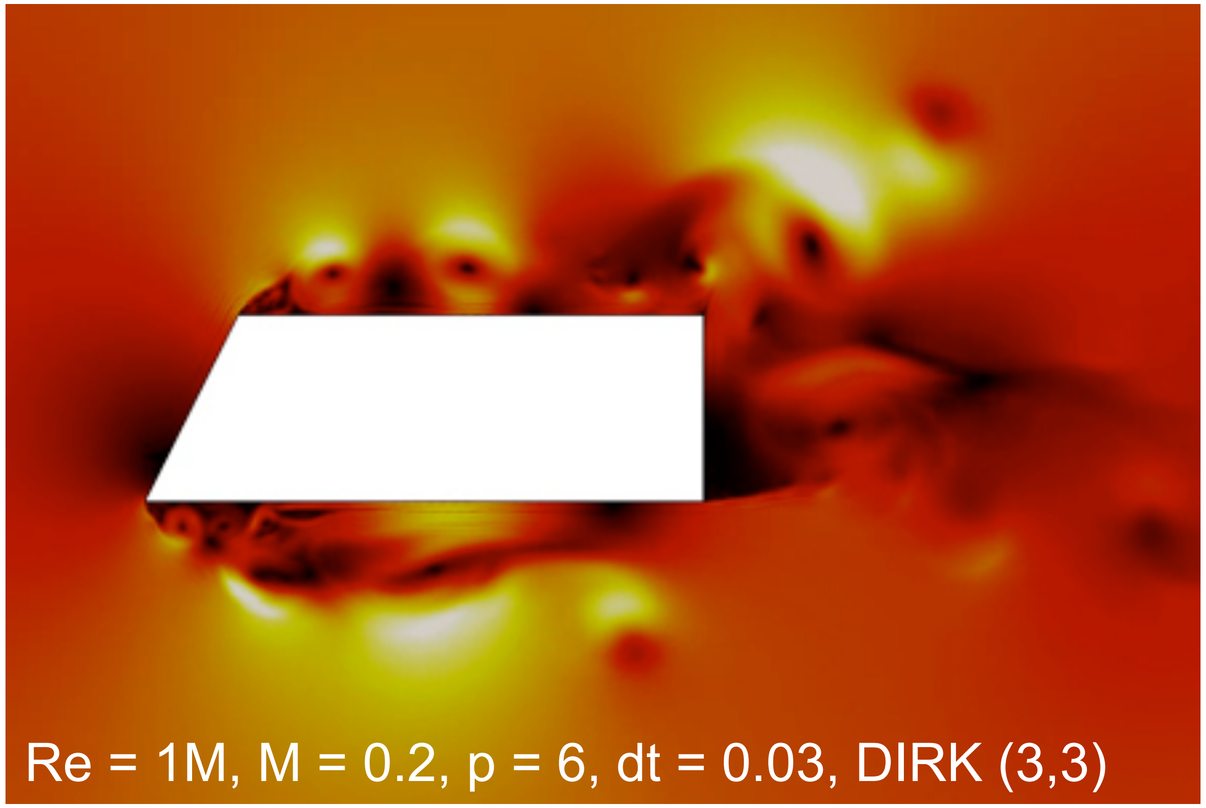




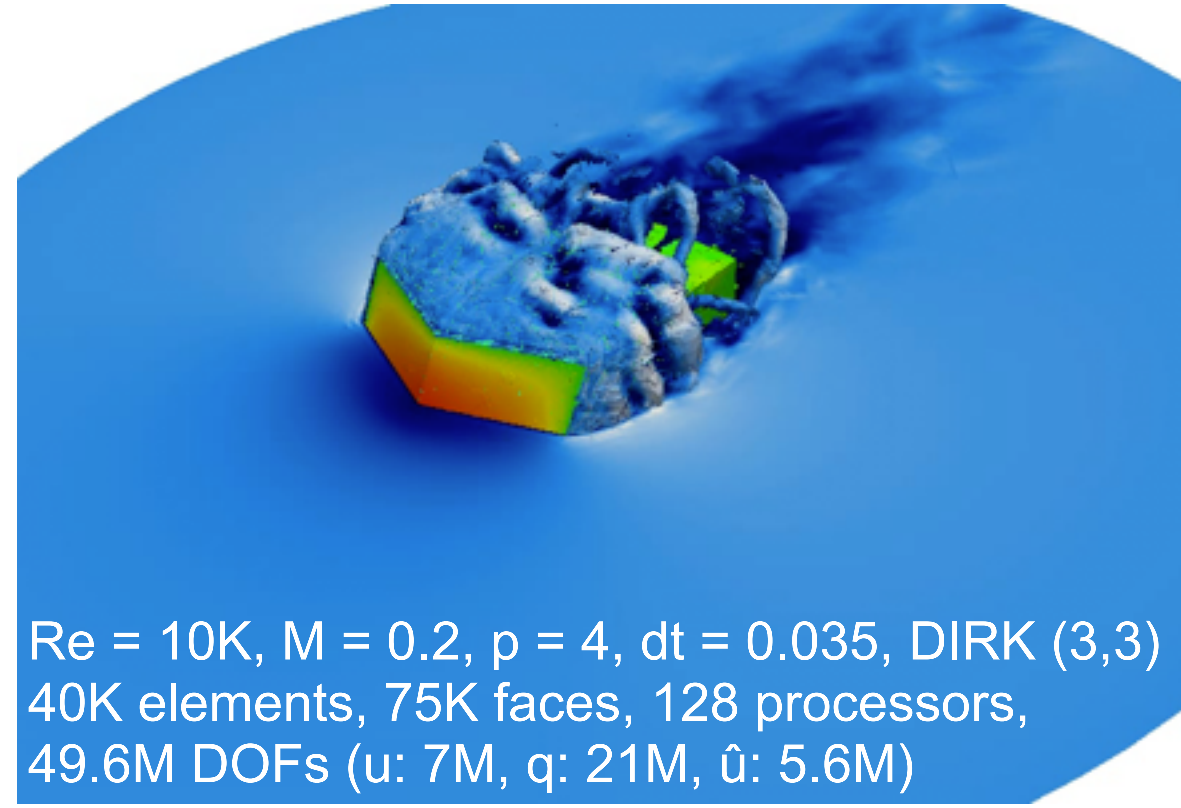
same curved boundaries & mesh topology

Predict sound spectrum: boundary layer meshes

with N.C. Nguyen & J. Peraire

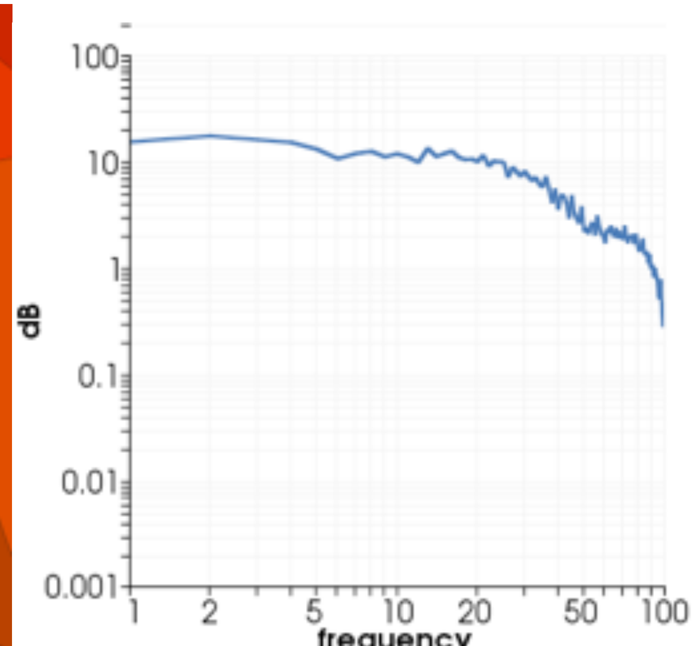


Non-resolved boundary layer: artificial recirculation !!

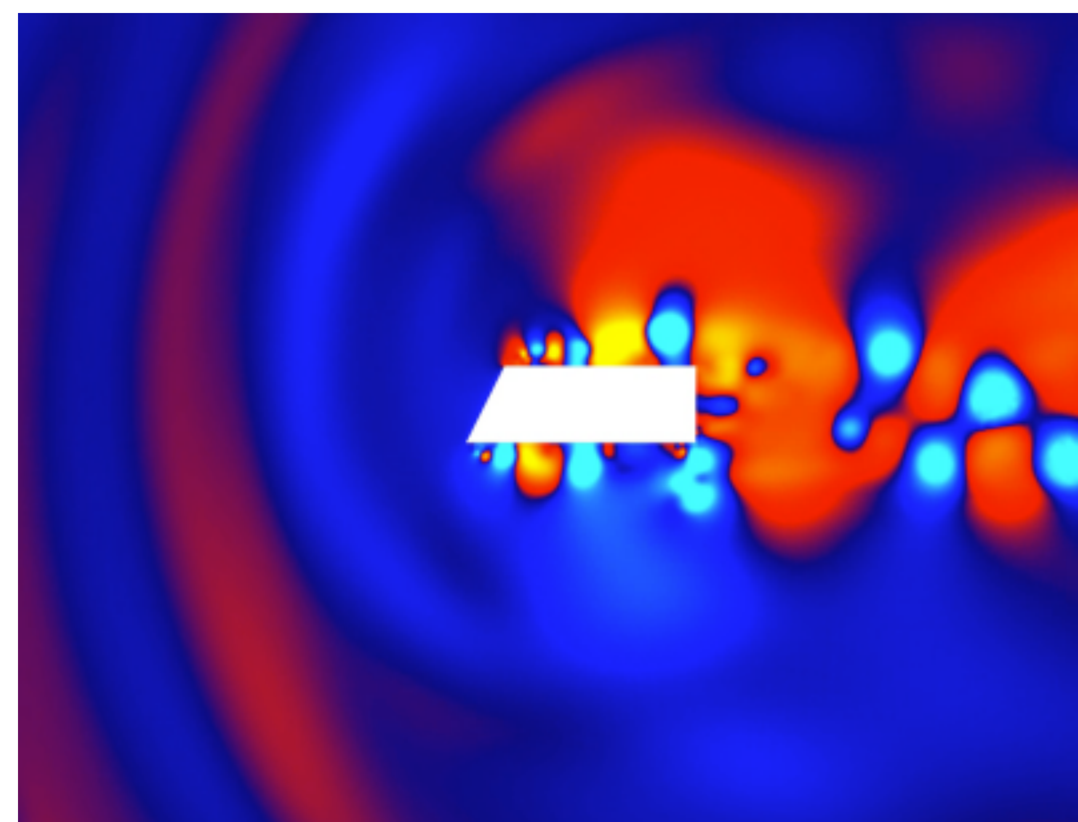


Pressure on the panels and density iso-surface

Curved boundary layers: all-acute-tetrahedra



Sound spectrum

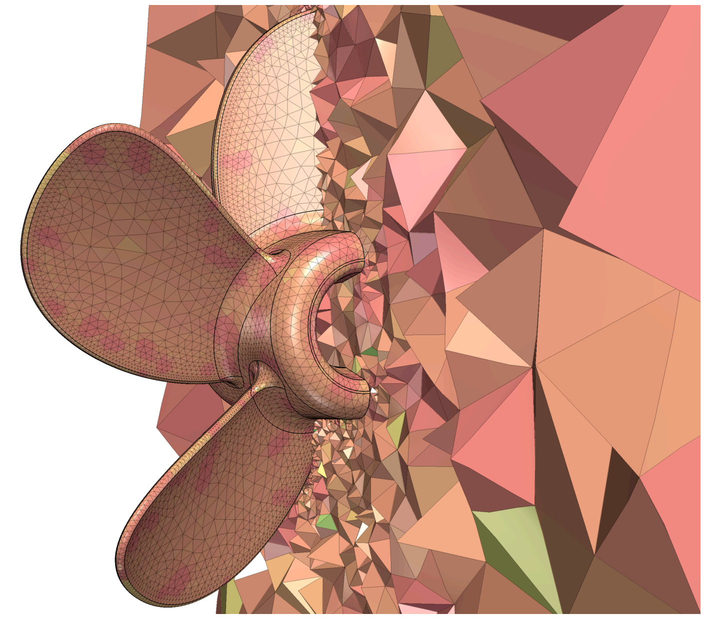


High-order ILES: captures pressure perturbations

Meshing work flow: geometry representation

Simulation domain
(geometry representation)

Meshing



Polytopal approximation
(mesh)

GEOMETRY SOURCE → GEOMETRY FOR SIMULATION → MODEL FOR SIMULATION → MESH FOR SIMULATION

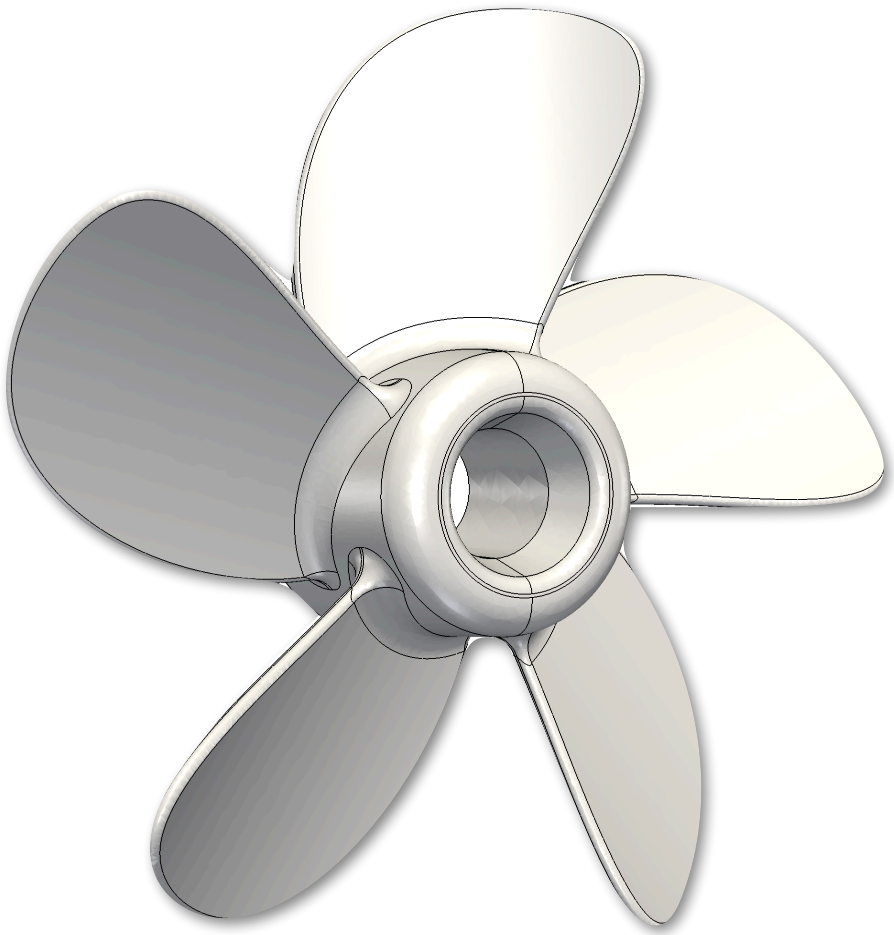
Preparing geometry for simulation:
- Segmentation
- Healing
- De-featuring

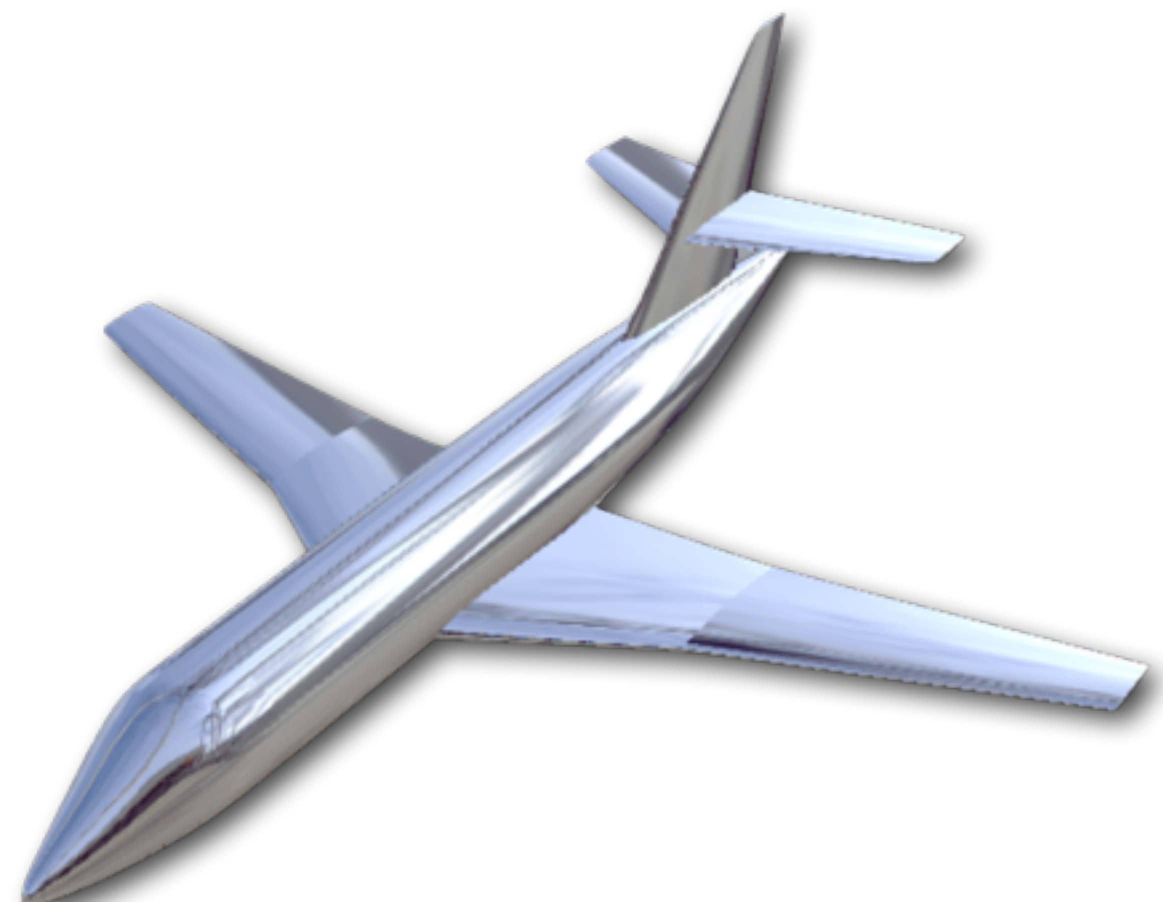
Assign boundary, materials, sources, …

Choose element sizes & types, mesh algorithms …

Geometry representation: CAD b-rep

- **CAD boundary representation**: a planar (volumetric) geometry is represented by a finite number of vertices and curves (vertices, curves and surfaces) that describe the boundary curves (surfaces).

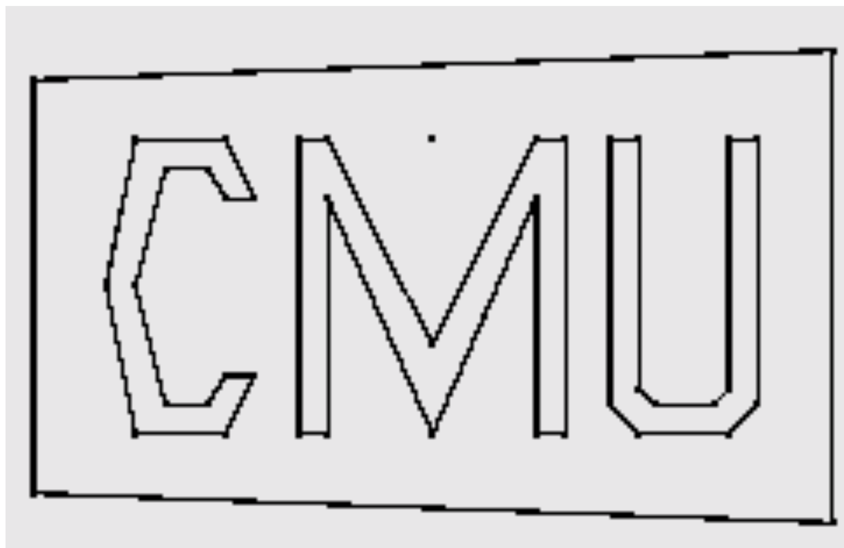




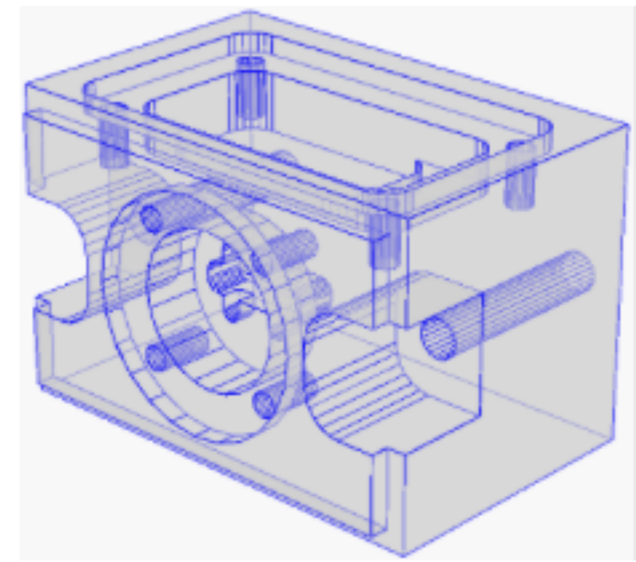
CAD b-rep for a propeller and a Falcon aircraft

Geometry representation: tesselation

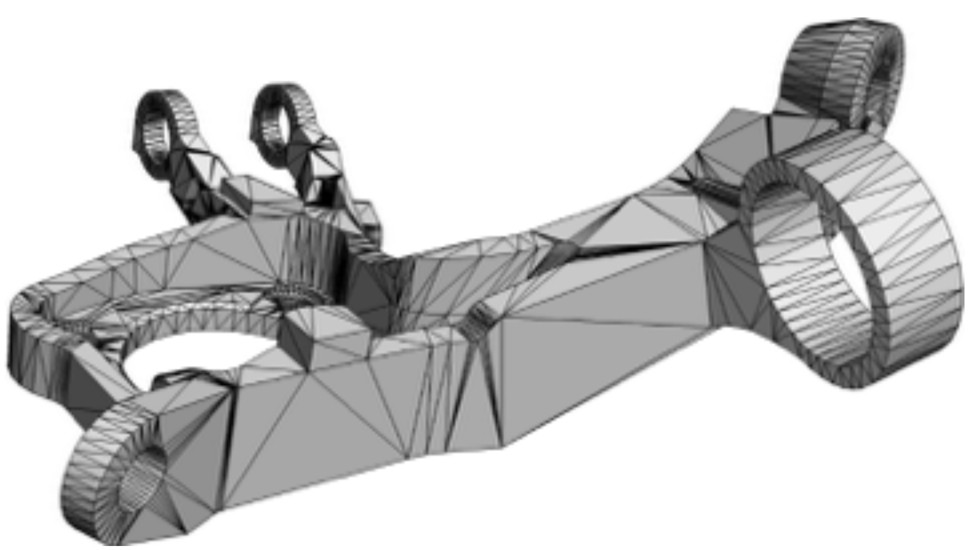
- **Tessellation**: a planar (volumetric) geometry is represented by a finite set of segments (polygons) that compose the boundary curves (surfaces).



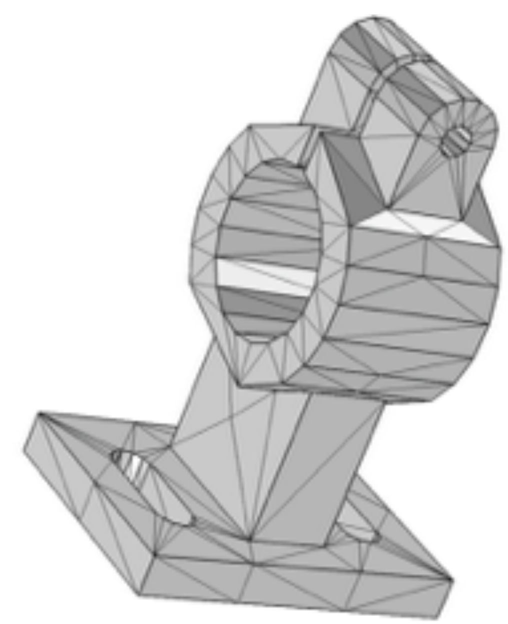
from https://www.cs.cmu.edu/~quake/triangle.demo.html



from tetgen.org



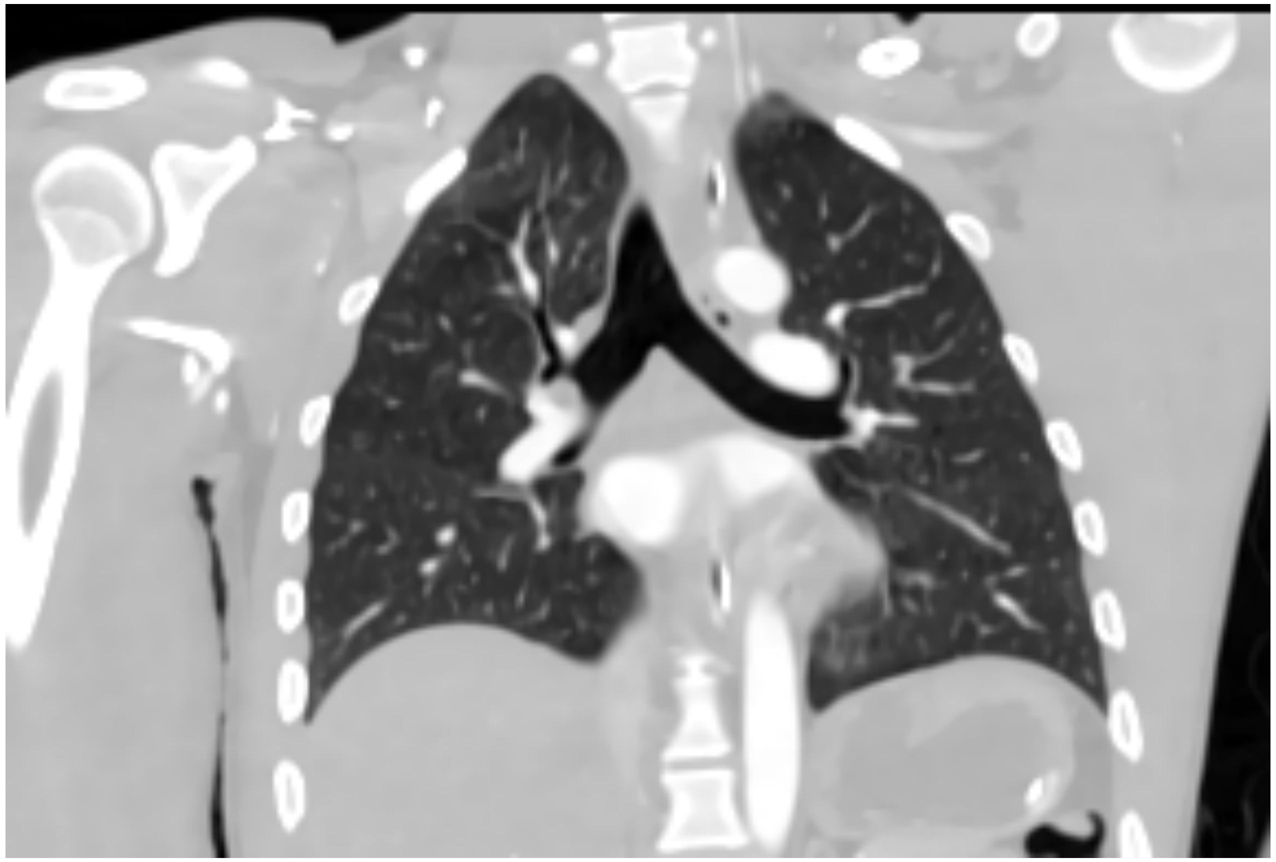
from www.meshgems.com



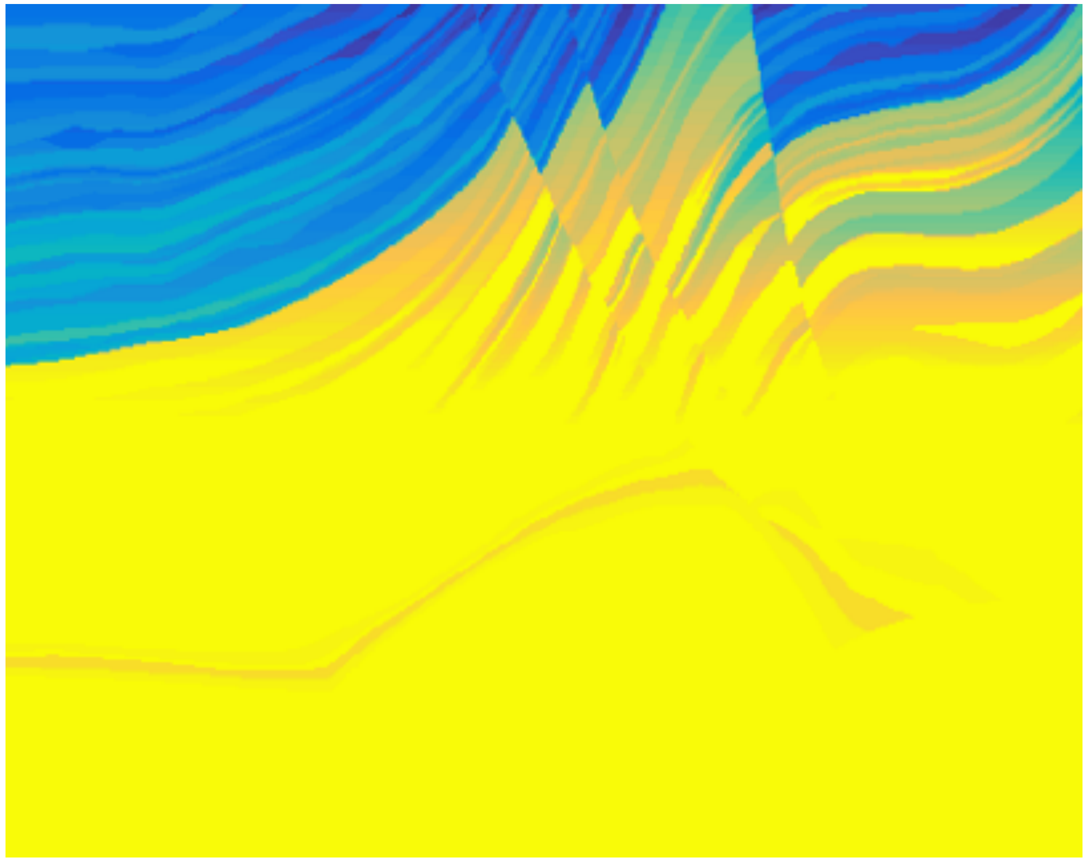
from www.comsol.com

Geometry representation: images

- **Images**: a planar (volumetric) geometry is represented by the intensity of the pixels (voxels) of a 2D (3D) image.

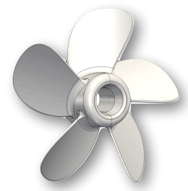
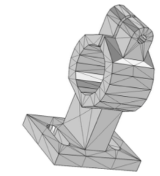
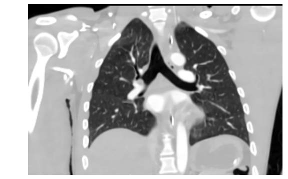


Slice of a 3D Image including the lungs (from Slicer3D)

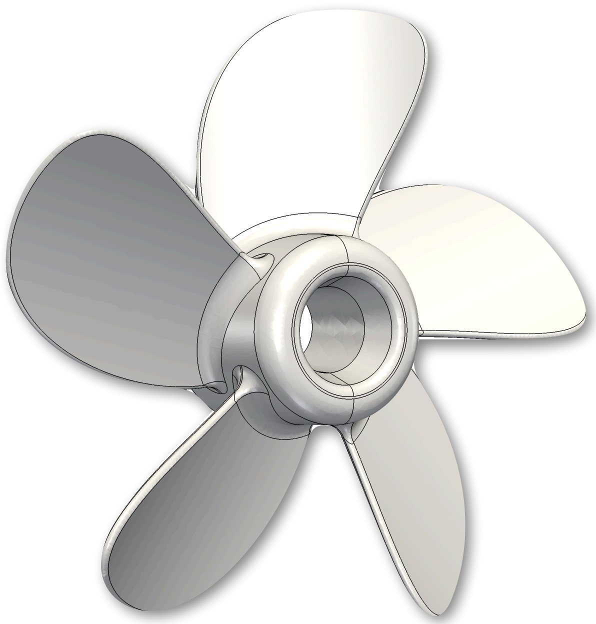


2D image (velocities) of a geological formation

Geometry representation: comparison

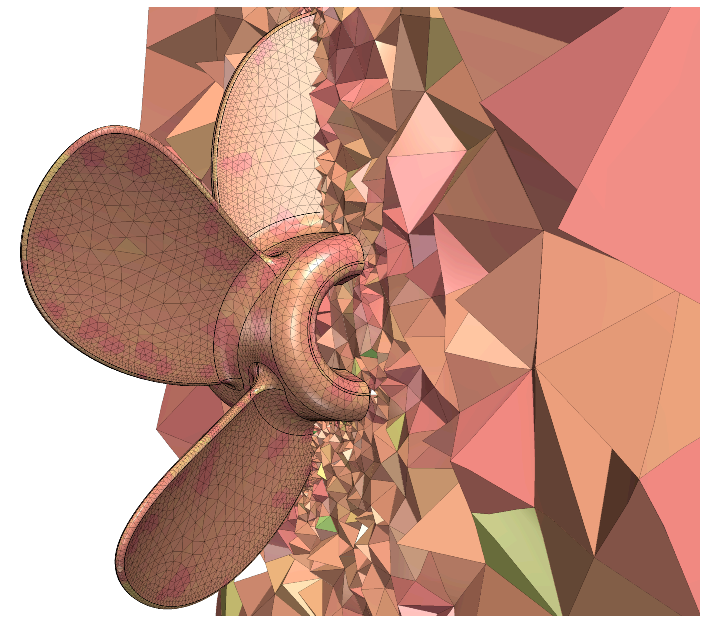
	CAD b-rep 	Tessellation 	Image 
Adequacy	Industrial designs	3D printing, CAM, scanning (from object, CAD, or image)	2D (3D) imaging (MRI, tomography, …)
Applications	Mechanical, aeronautical, …	Legacy data	Bio-medical, geophysical, …
Boundary conditions	Applied to vertices, curves, surfaces	Applied to groups of points, segments, & polygons	Applied to a segmentation (e.g. tesselation)
Curved entities	Yes, through free-form curves (surfaces)	No, requires several segments (polygons)	No, requires several pixels (voxels)
Accuracy	Piece-wise polynomial (B-splines) or rational (NURBS)	Piece-wise linear	Pixel (voxel) constant
Smoothness	Allows continuity of geometry, normals, curvatures, …	Continuity of geometry	Discontinuous
Number of Entities	#vertices + #curves (+ #surfaces)	#vertices + #segments (+ #polygons)	width x height (x depth)
Conics	Exact representation	Piece-wise linear approximation	Pixelized (voxelized)
Formats	IGES, STEP, …	STL, …	DICOM, TIFF, PNG, …
Representation issues	Import / export inconsistencies	Preserving sharp features	Noise
Preparation issues	Not always water-tight	Not always water-tight	Requires segmentation

Meshing work flow: preparing geometry for simulation



Simulation domain
(geometry representation)

Meshing



Polytopal approximation
(mesh)

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Preparing geometry for simulation:

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- Healing
- De-featuring

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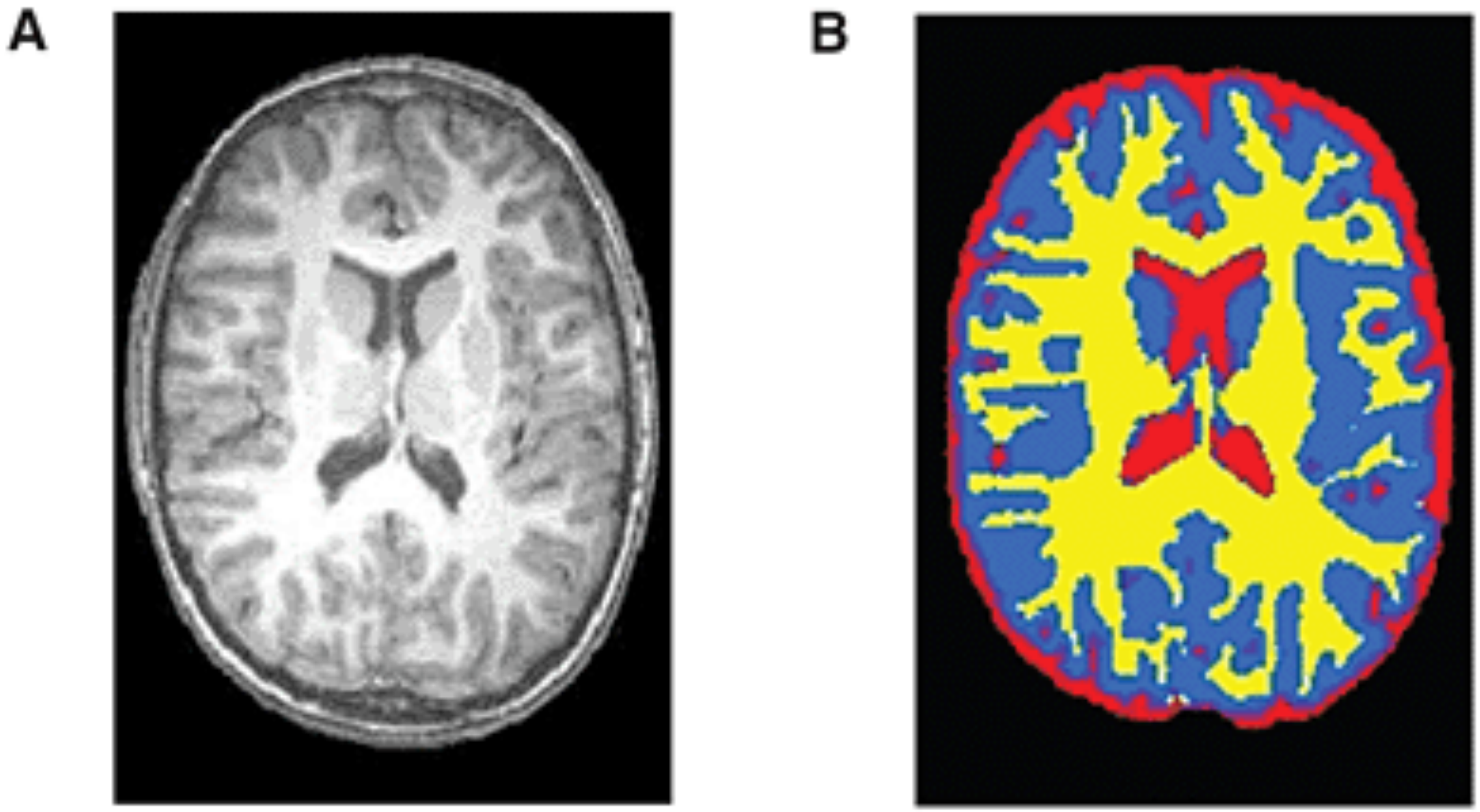
Choose element sizes & types, mesh algorithms …

Geometry for simulation: segmentation

- **Segmentation** (image-to-tessellation): group the pixels (voxels) that determine the areas (volumes) of interest.



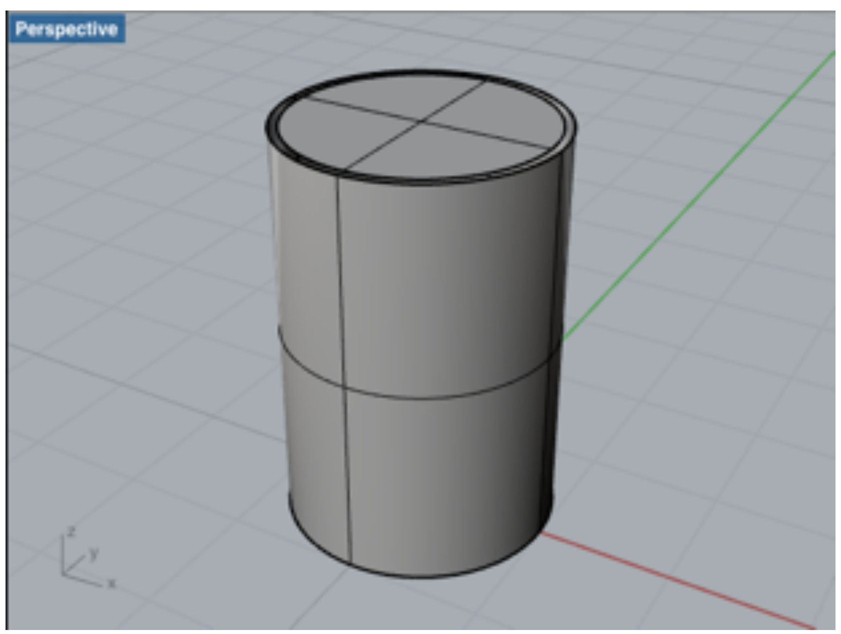
Slice of a 3D brain image and segmentation in three volumes

from http://pubs.niaaa.nih.gov/publications/arh313/243-246.htm

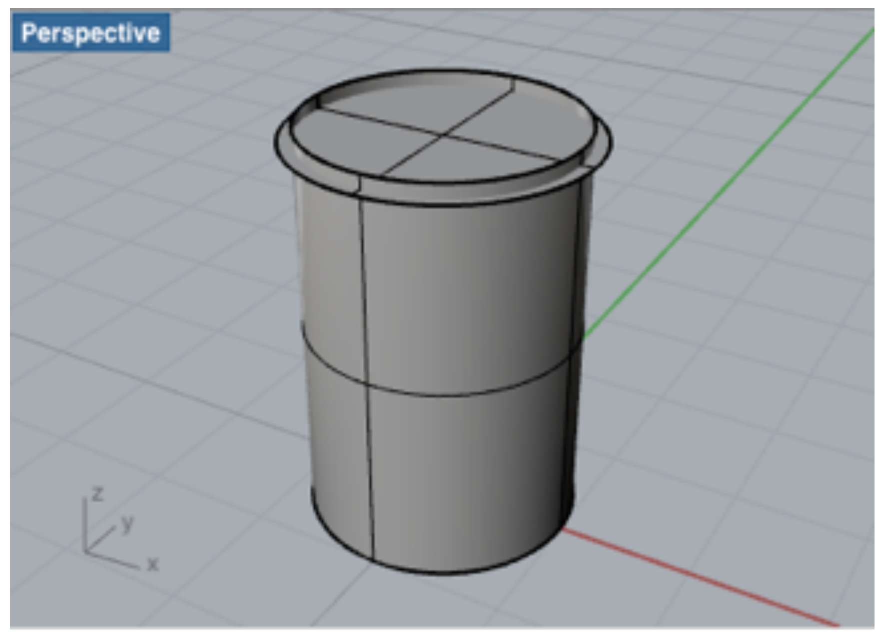
Geometry for simulation: healing

- **Healing** (CAD, tessellation): the boundary representation has to determine properly the interior and exterior of the areas (volumes).

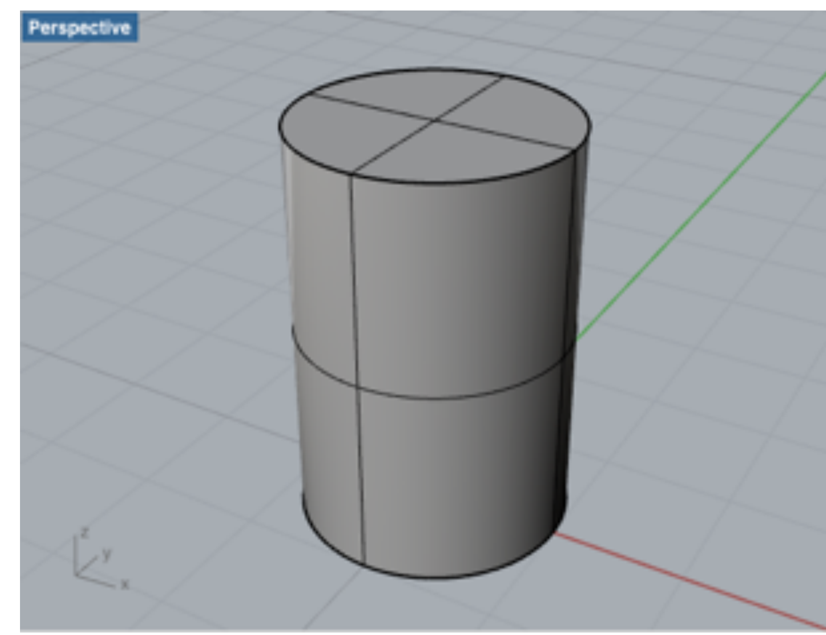
- **Watertight**. Informally, areas (volumes) would not leak water.



Non-watertight cylinder (leaks):
- Surface gaps
- Surface limits do not match



Non-watertight cylinder (leaks):
- Surface intersections
- Curve limits do not match

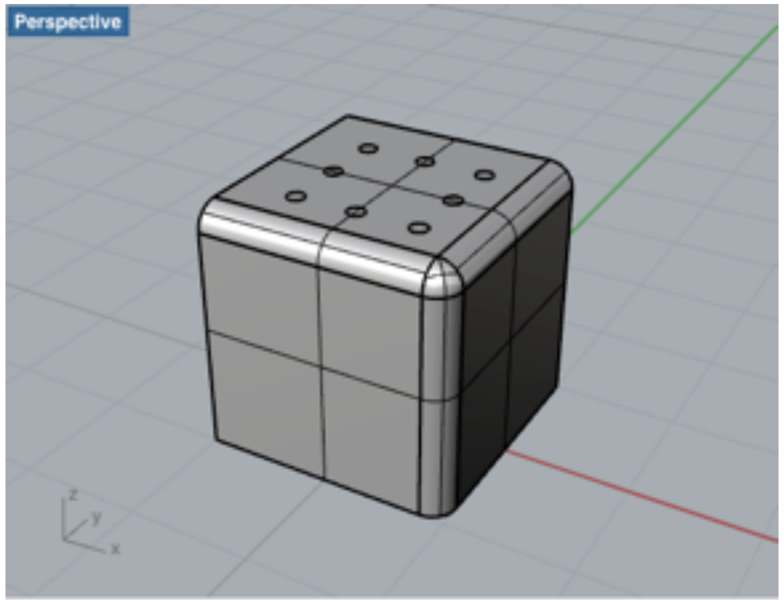


Watertight cylinder (no leaks):
- Closed volume
- Surface limits match

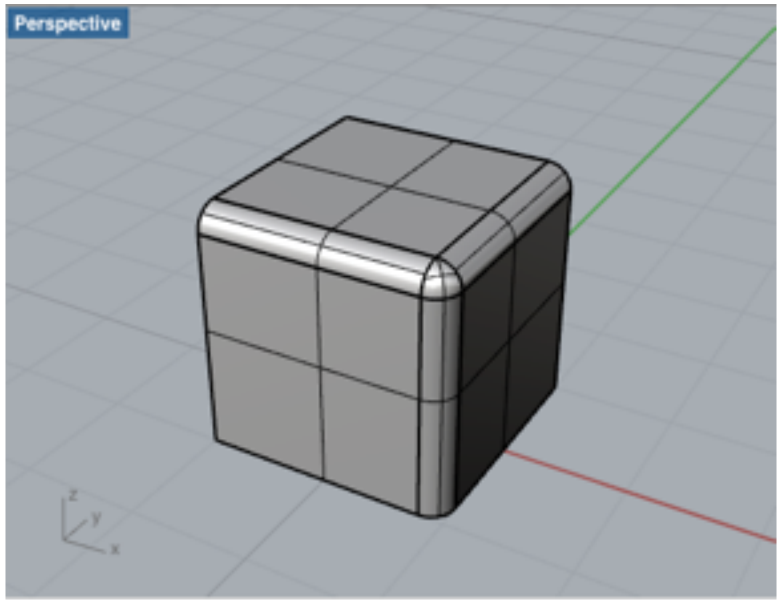
- **Noise**. Geometry and / or topology have random artifacts
  - Bumpy curves (surfaces), disconnected areas (volumes), …

Geometry for simulation: de-featuring

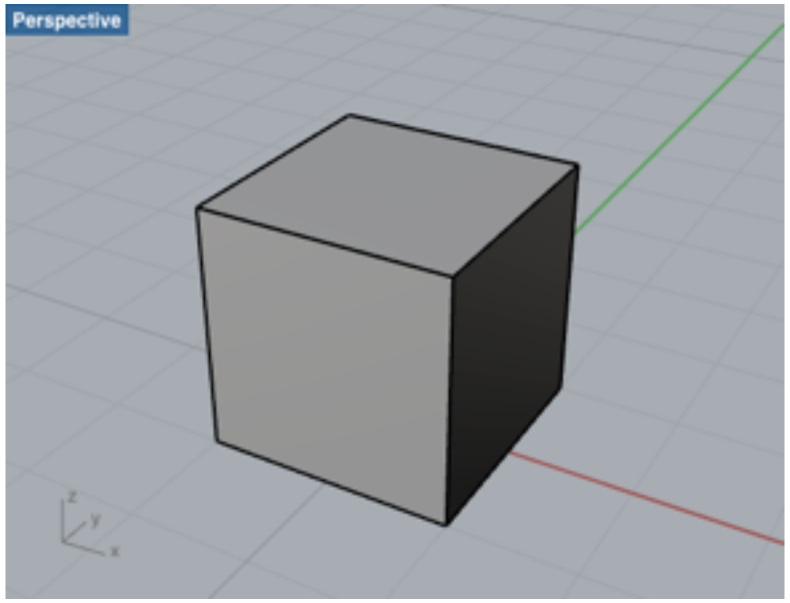
- **De-featuring** (CAD, tessellation, image): remove excessive detail not required for the simulation.
- De-featured geometry contains only the **details relevant for the simulated physics**.



Small extrusions



Fillets



Simplified geometry

Removing features non-relevant for the simulation

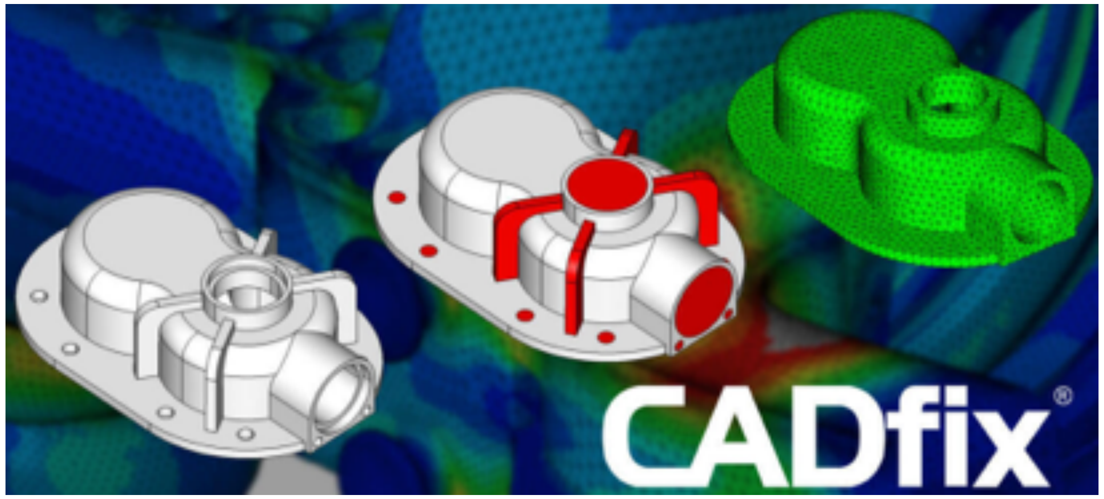
Geometry for simulation: designed vs. digitized

- Geometry preparation issues can be inherent to:
  - representation but also,
  - obtention process (designed / digitized)

Issues		Designed (CAD, tesellation)	Digitized (tesellation, image)
Healing		X	X
	Gaps	X	
	Intersections	X	
	Holes		X
Noise			X
	Topological		X
	Geometrical		X
Segmentation			X (Only for images)
Defeaturing		X	X

Geometry preparation software: CAD b-rep

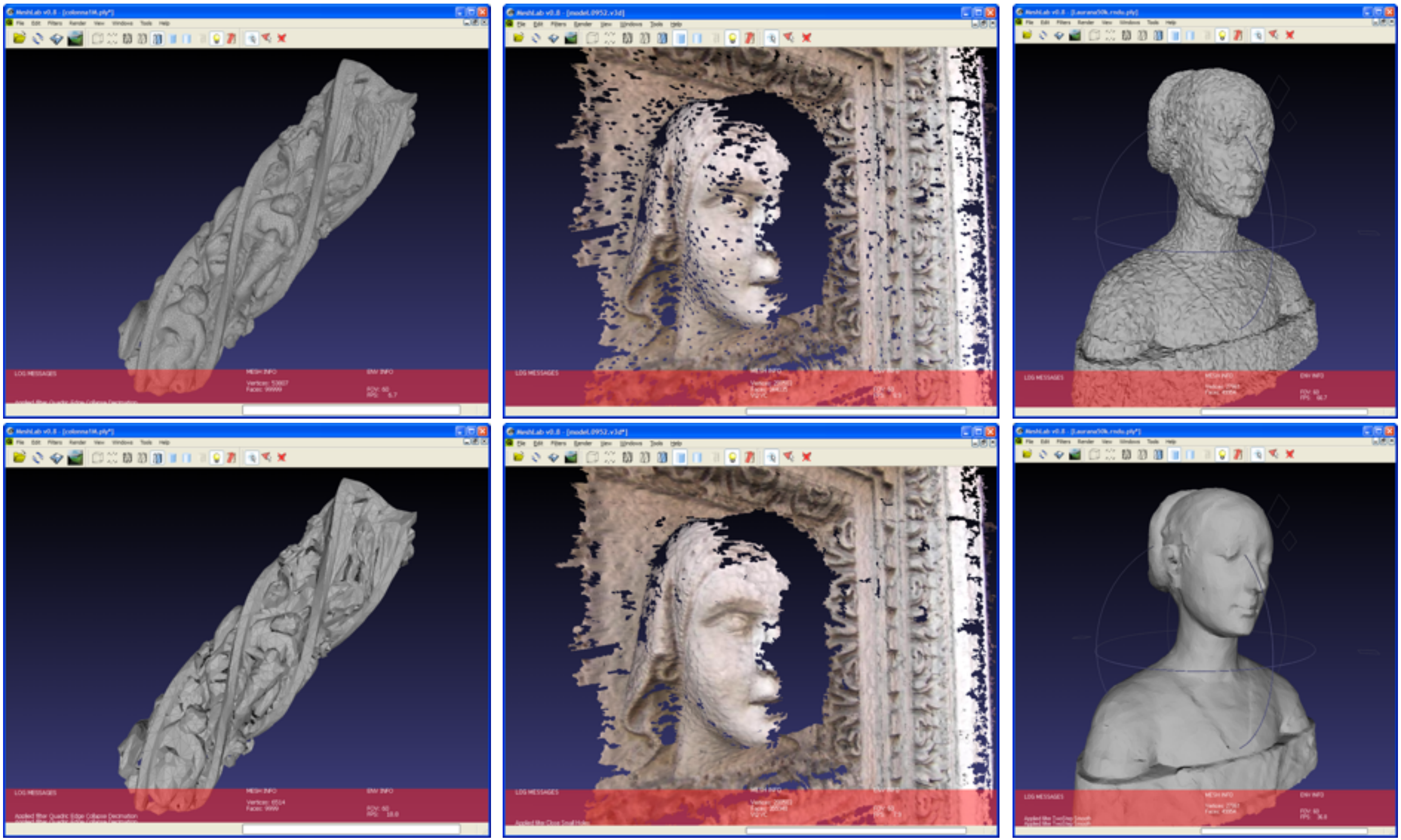
- **Solidworks**: parametric featured-based solid modeller
  http://www.solidworks.com
  - parameters: radius, height, relative distance, angle, …
  - features: extrusions, rounding, holes, chamfer, …
- **Rhinoceros**: excels in free-form surface modelling with NURBS. It provides healing operations (remove leaking vertices / curves)
  http://www.rhino3d.com
- **CADfix**: excels in import, heal, de-feature, and export geometry.
  http://iti-global.com/cadfix



In red features to be removed
In green the mesh on the de-featured model

Geometry preparation software: tessellation

- **Meshlab (free)**: prepare unstructured 3D triangular meshes
  http://meshlab.sourceforge.net

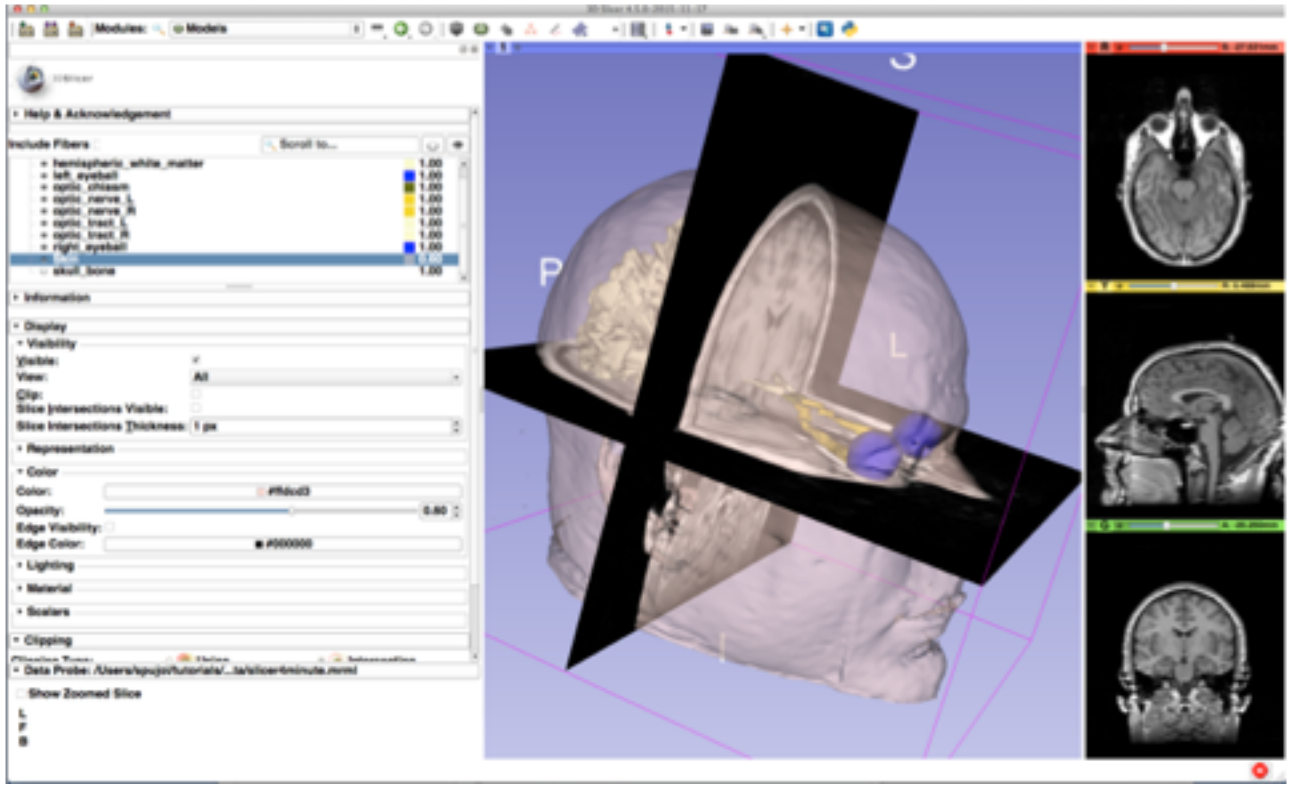


Edge collapse simplification | Hole filling | Geometry noise removal

Geometry preparation software: images

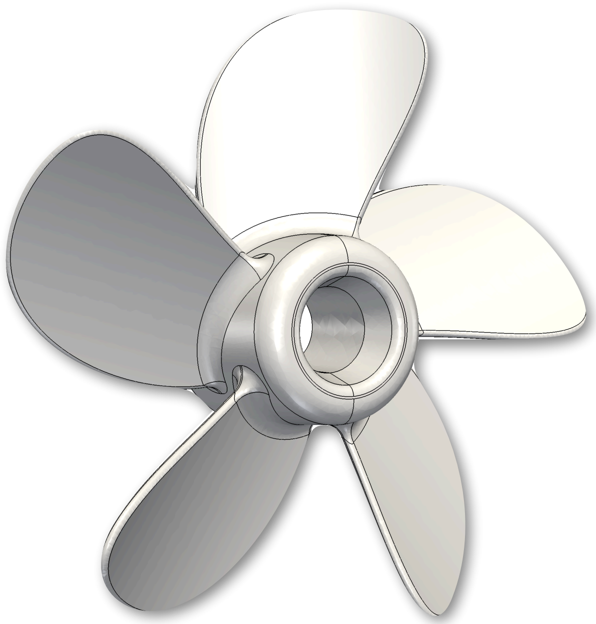
- **Amira**: life-sciences image edition and segmentation with automatic / semi-automatic / manual tools. It also features tetrahedral mesh generation
  http://www.fei.com/software/amira-3d-for-life-sciences/

- **3D Slicer** (free): tools for automatic / semi-automatic / manual segmentation of medical imaging. Extendable with plug-ins.
  http://meshlab.sourceforge.net

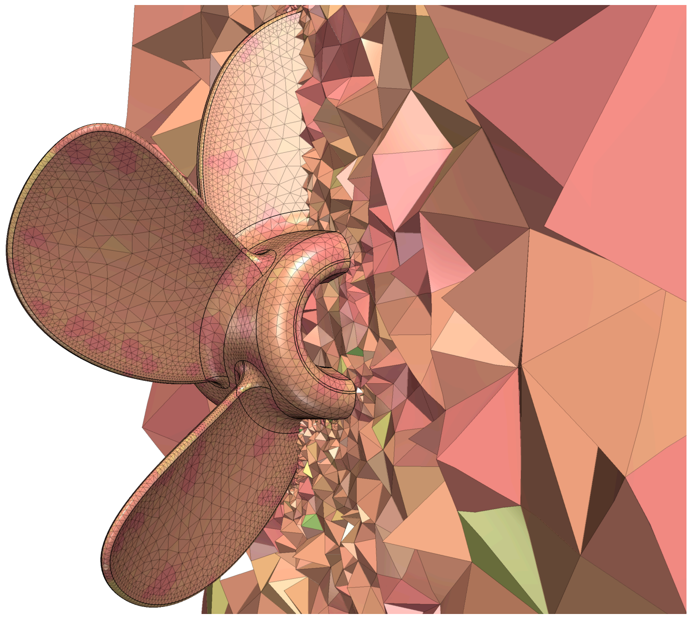


3D Slicer GUI

Meshing work flow: meshing set up & algorithms



Meshing



Simulation domain
(geometry representation)

Polytopal approximation
(mesh)

GEOMETRY SOURCE → GEOMETRY FOR SIMULATION → MODEL FOR SIMULATION → MESH FOR SIMULATION

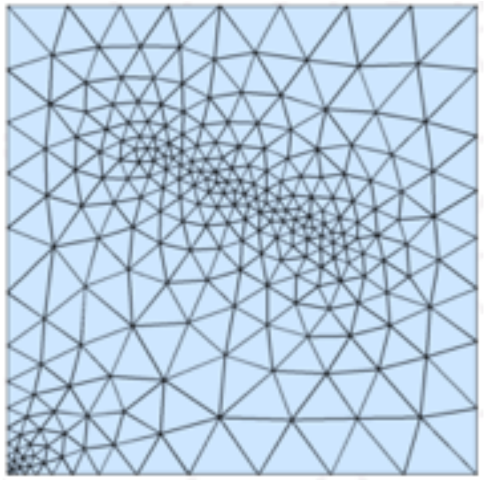
Preparing geometry for simulation:
- Segmentation
- Healing
- De-featuring

Assign boundary, materials, sources, …

Choose element sizes & types, mesh algorithms …

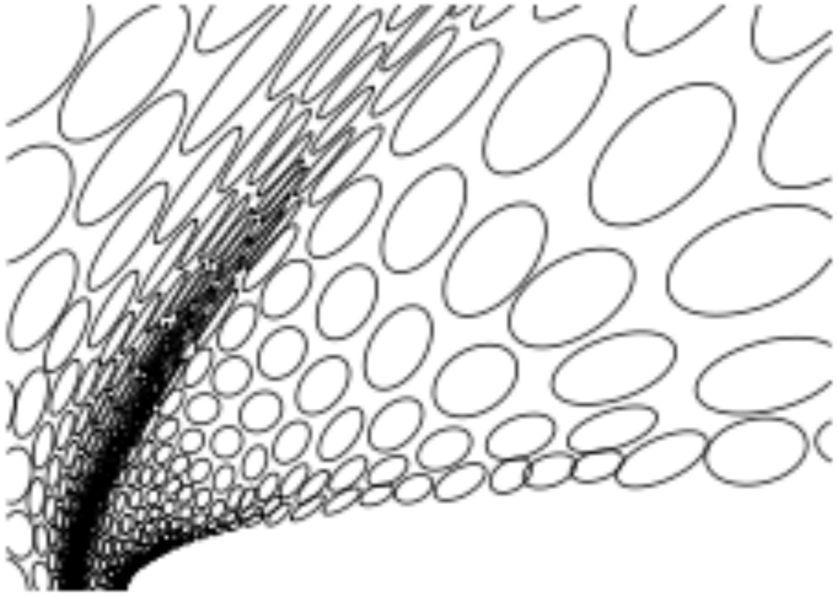
Adaptive meshing: non-uniform size (and shape)

- **Isotropic meshing**. From a size field on a mesh to a mesh with elements of varying size and similar shape.

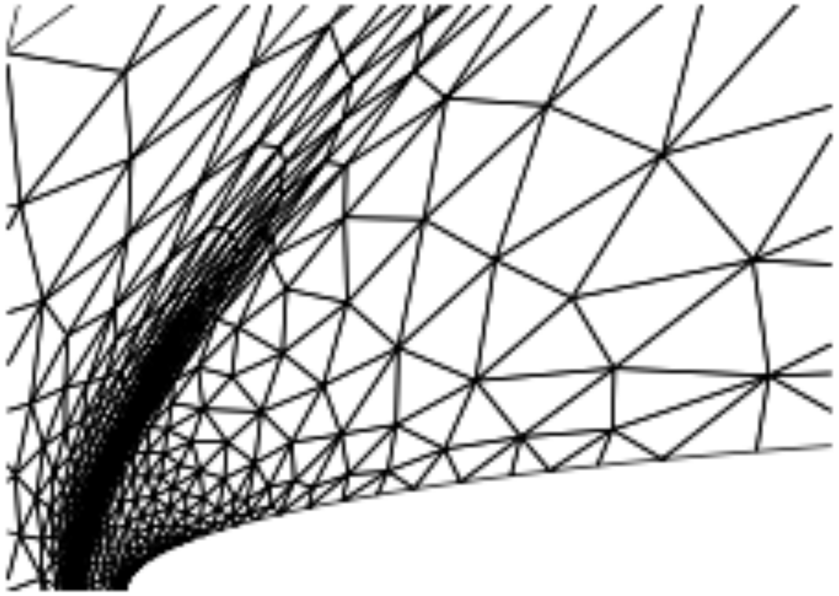


from http://persson.berkeley.edu/distmesh/

- **Anisotropic meshing**. From a metric field on a mesh to a mesh with elements of varying size and aspect ratios.



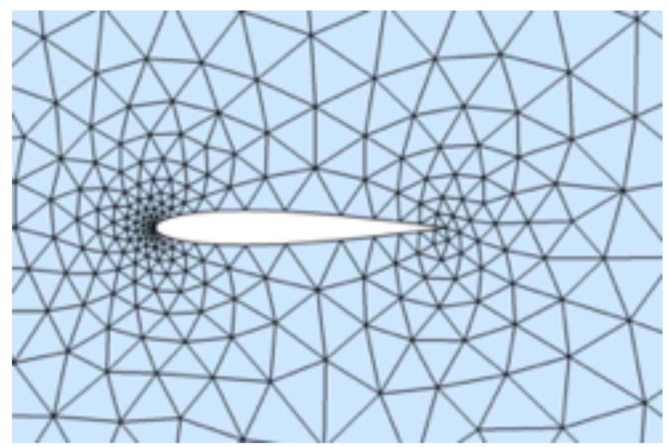
Metric field



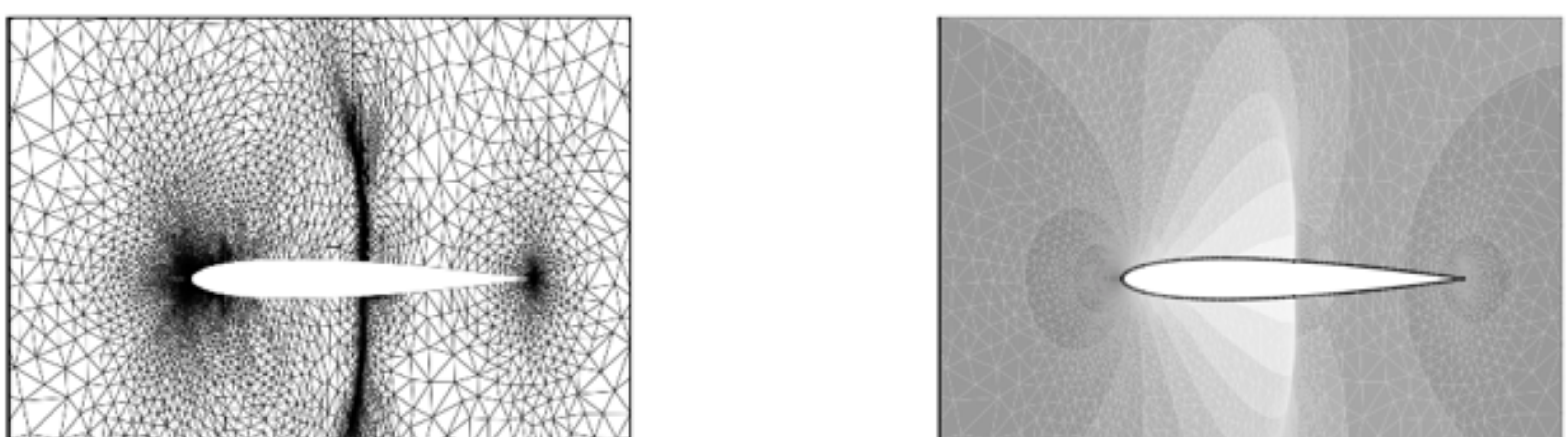
Anisotropic triangular mesh
from Yano'12

Adaptive meshing: non-uniform size (and shape)

- Reduce the computational cost by reducing the number of DOFs
- Or, obtain the maximum accuracy for a given cost
  - **Distmesh** (implicit geometry). From a distance function and size sources to a triangular mesh with non-uniform sizes.
    http://persson.berkeley.edu/distmesh/



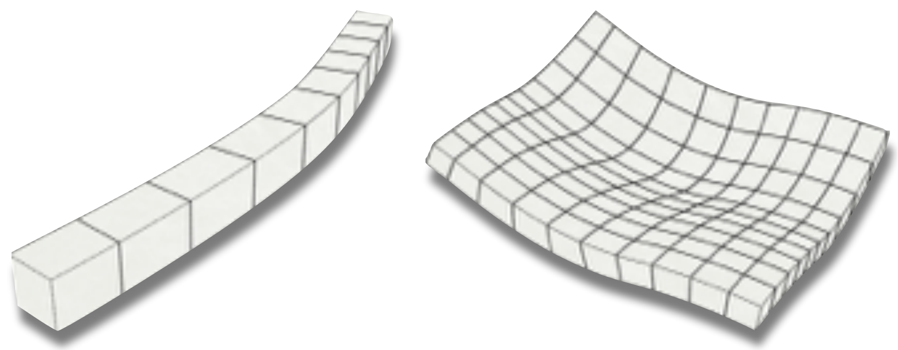
  - **BAMG** (previous mesh). From a size / metric field on a background mesh to an anisotropic triangular mesh.
    http://www.ann.jussieu.fr/~hecht/ftp/bamg/



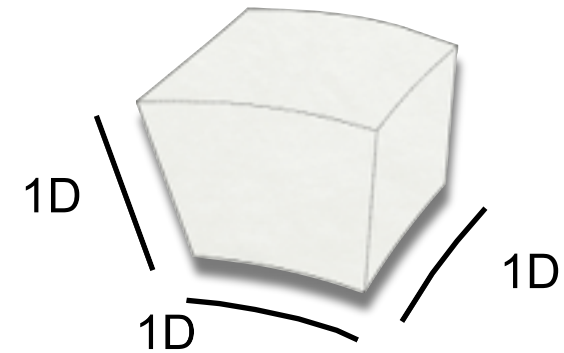
Iteration 19, 12057 vertices and 23756 triangles, hmin = 0.000576461 Iso Mach

Element types: hexahedra (quadrilaterals)

- **Exploitable structure:**



Face against face: stacks and layers



Tensor product: 1D x 1D x 1D

- **Align stretched hexahedra with solution features:**
  - boundary layer, composites, …
- **Fast unstructured hexahedral solvers**:
  - **Spectral Element Method:** diagonal mass matrix [Patera’84] [Fischer'97]
  - **Line Discontinuous Galerkin**: sparsity of finite differences [Persson’13]
- **Automatic unstructured hex-meshing: open problem !!**

[Blacker'00] [Tautges’01] [Staten *et al.*'10] [Ledoux & Shepherd’10] [Roca & Sarrate’10]

Element types: tetrahedra (triangles)

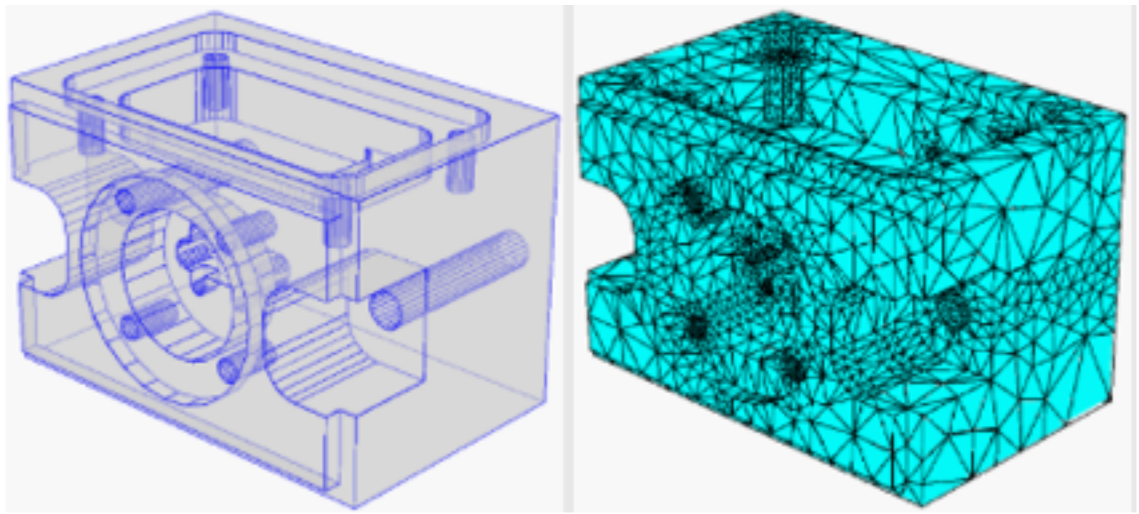
- **Flexible structure**

Face against vertex: no stacks, no layers

  - **Refine and coarsen easily even with stretching**
  - **Complete set of basis functions**
- **Unstructured tetrahedral solvers are slower but,**
- **High geometrical flexibility:** automatic tetrahedral (triangular) meshing for complex geometries with **mature technologies**
  - Delaunay, advancing front, overlay grid

Delaunay meshing (points / tessellation)

- **Delaunay Tessellation**. Interior of each triangle (tetrahedron) circumcircle (circumsphere) does not contain any point.
  - Maximizes the minimum angle.
- **Constrained Delaunay Tessellation (CDT)**. Almost Delaunay Tessellation of an area (volume) that preserves the domain boundary determined by a given tessellation

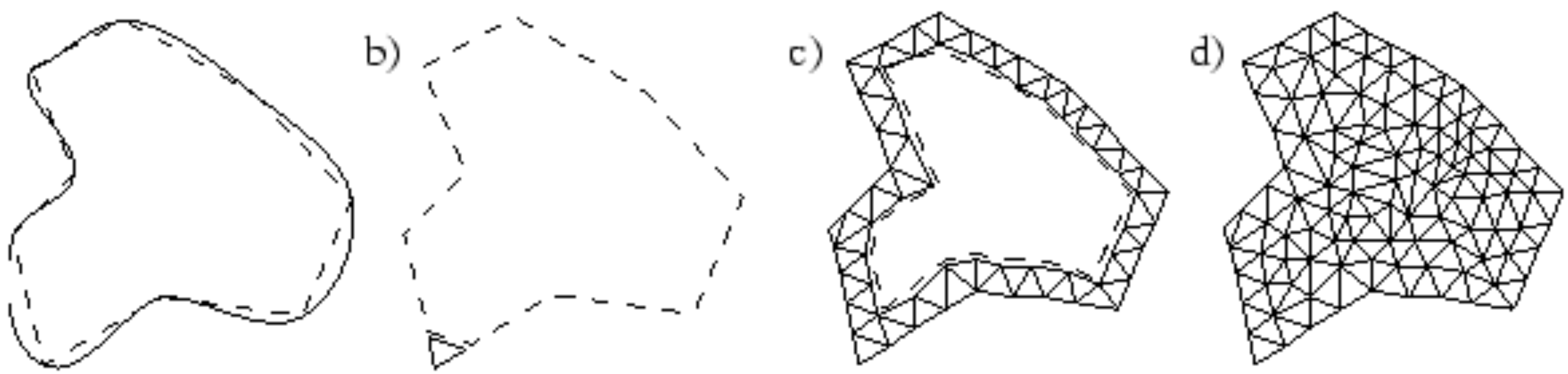


Boundary tessellation   CDT mesh

  - **Triangle** (free): b-rep tessellation to triangular mesh (Delaunay) https://www.cs.cmu.edu/~quake/triangle.html
  - **Tetgen** (free): b-rep tessellation to tetrahedral mesh (Delaunay) http://wias-berlin.de/software/tetgen

Advancing front (b-rep tessellation)

- The triangles (tetrahedra) are generated layer by layer starting from the boundary and finishing in the inner part of the area (volume).
  - High element quality next to the boundary, where physical conditions are imposed.

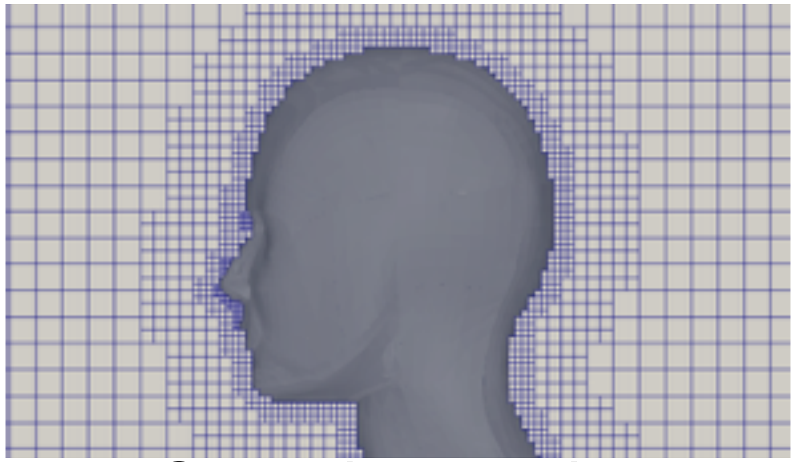


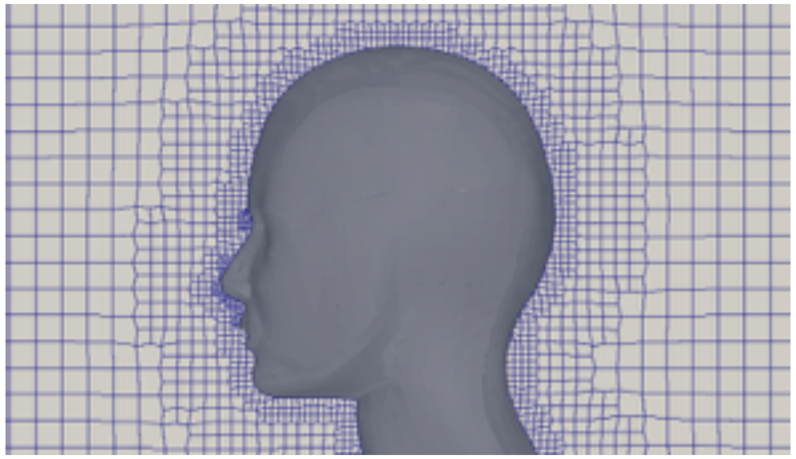
from http://www.michael-burghardt.de/diss/node74.html

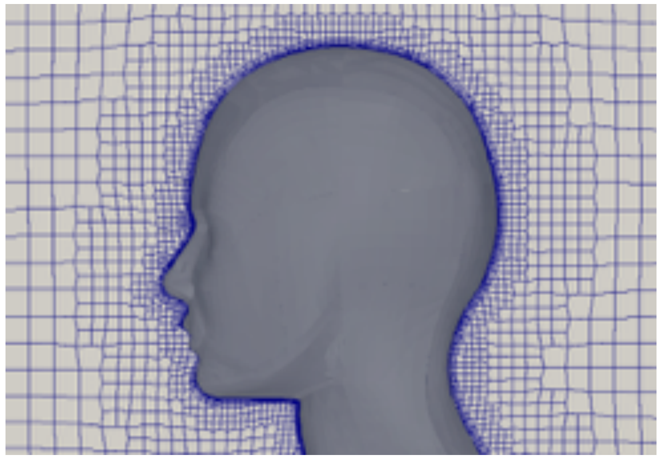
- **Netgen**: CAD or STL to surface (tris) & volume (tets) mesh
  https://sourceforge.net/projects/netgen-mesher/

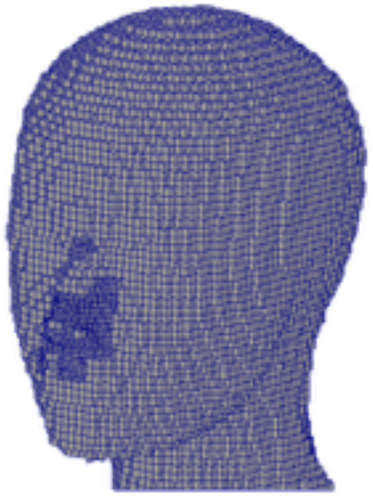
Overlay grid meshing (CAD and tessellation)

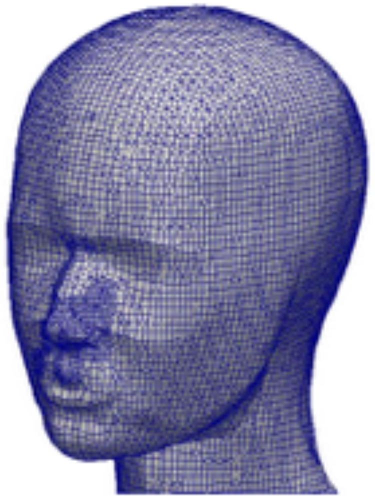
- Cuts an adapted cartesian mesh with the input b-rep
  - Really robust approach


Cartesian mesh


Snap to surface


Insert boundary layer


Staircase effect

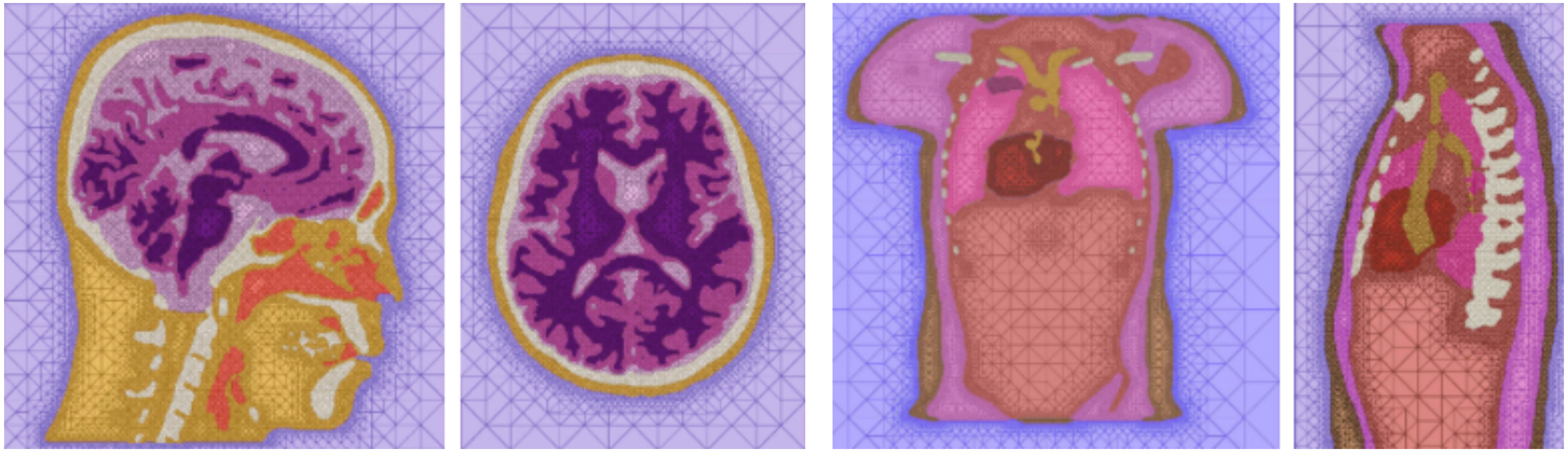

Lower quality at boundaries

from https://openfoamwiki.net/images/f/f0/Final-AndrewJacksonSlidesOFW7.pdf

snappyHexMesh

Overlay grid (segmented image)

- Cuts and adapted cartesian mesh with the different regions of a segmented image
  - **Cleaver** (image): tetrahedral meshes that conform approximately the physical boundaries of multiple volumes https://www.sci.utah.edu/cibc-software/cleaver.html

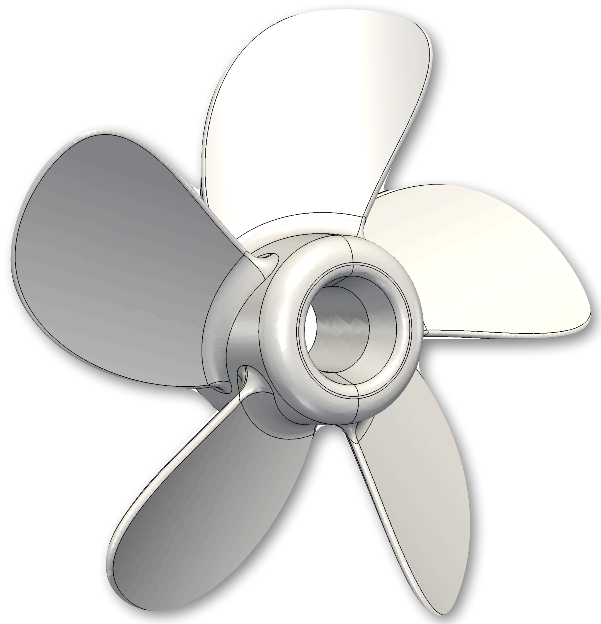


Multivolume meshes for the head and torso
from http://www.cs.utah.edu/~bronson/papers/Cleaver_IMR_2012.pdf

Meshing methods

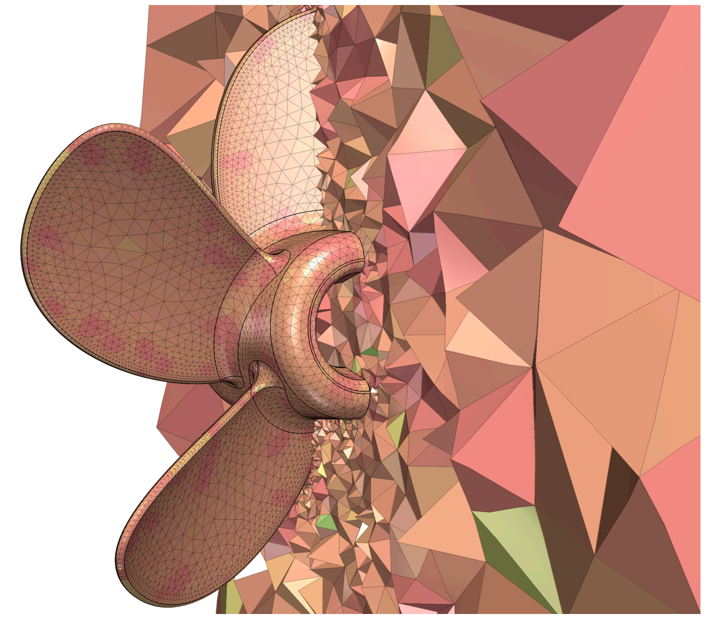
	Delaunay	Advancing front	Overlay grid
Conforming mesh	Yes	Yes	Not always
Boundary conformal	Yes	Yes	Approximately
Quality of boundary elements	Good	Maximum	Low
Insensitive to rigid motions	Yes	Yes	No
Deals with "dirty" geometry (Robustness)	Good	Good	Maximum

Meshing work flow: integrated work flow



Simulation domain
(geometry representation)

Meshing



Polytopal approximation
(mesh)

GEOMETRY SOURCE → GEOMETRY FOR SIMULATION → MODEL FOR SIMULATION → MESH FOR SIMULATION

Preparing geometry for simulation:
- Segmentation
- Healing
- De-featuring

Assign boundary, materials, sources, …

Choose element sizes & types, mesh algorithms …

Integrated meshing work flow

- **Computer-Aided Engineering** (CAE): geometry, meshing, boundary conditions, simulation, post-processing results
  - **Gmsh** (free): excellent open source 3D mesh generator
    http://gmsh.info
  - **GiD** (CAD): powerful costumization of boundary conditions, materials, mesh file outputs, batch files, …
    http://www.gidhome.com
  - **Cubit** (CAD / tessellation): excels in hexahedral mesh generation
    https://cubit.sandia.gov
  - **Ansys ICEM / CFD** (CAD / tessellation): repair, tetrahedral meshing, overlay grid meshing, hybrid meshing, hexahedral meshing
    http://resource.ansys.com/Products/Other+Products/ANSYS+ICEM+CFD

What about our research in mesh generation

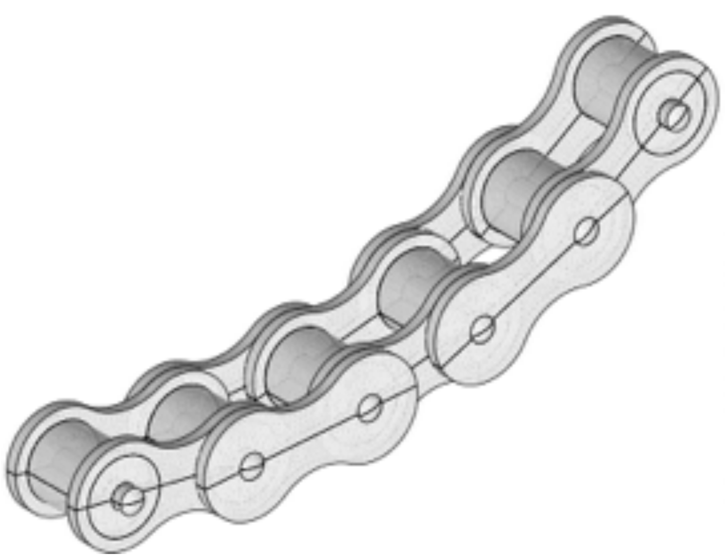
Discretizing (meshing) for high-fidelity simulation:

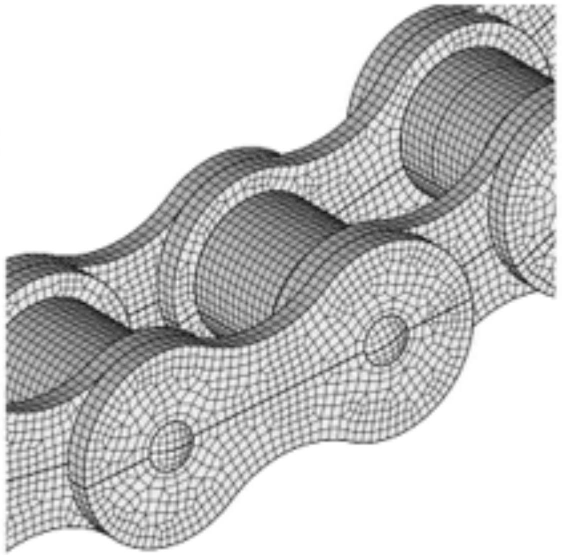
- geometrical fidelity
- physical fidelity
- numerical fidelity
- computational efficiency

CAD-based hierarchical meshing

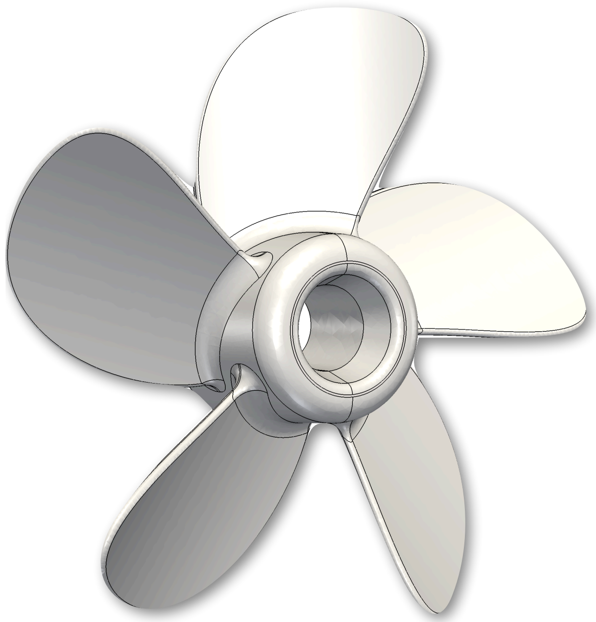
with E. Ruiz-Gironés, A. Gargallo-Peiró, A. Huerta, J. Peraire & J. Sarrate

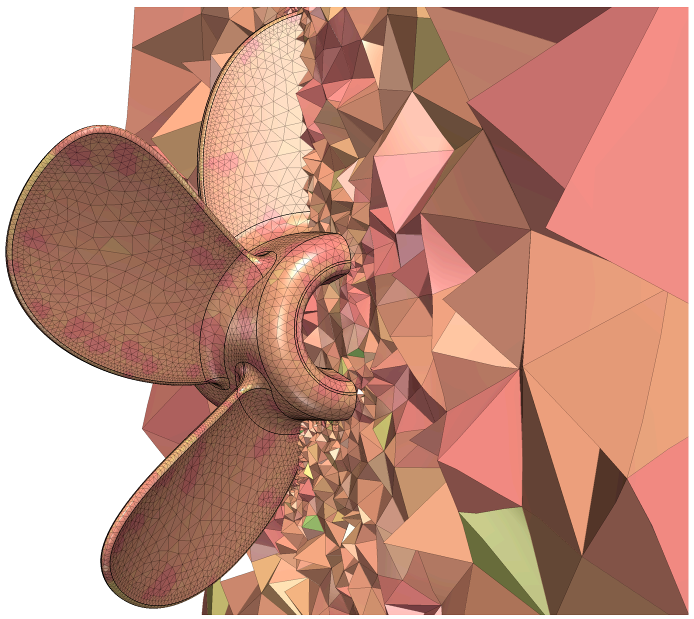
- **Problem**: approximate a domain with polyhedral elements
- **Solution**: from CAD to mesh by sorted discretization:
  - (1) vertices, (2) curves, (3) surfaces & (4) volumes
  - Met challenges: accuracy, complexity, curvature, sharp features …
  - and: industry standard, parameterizations, hierarchy of entities





with J. Sarrate & A. Huerta,
surface mesh projection, 2004, 2005



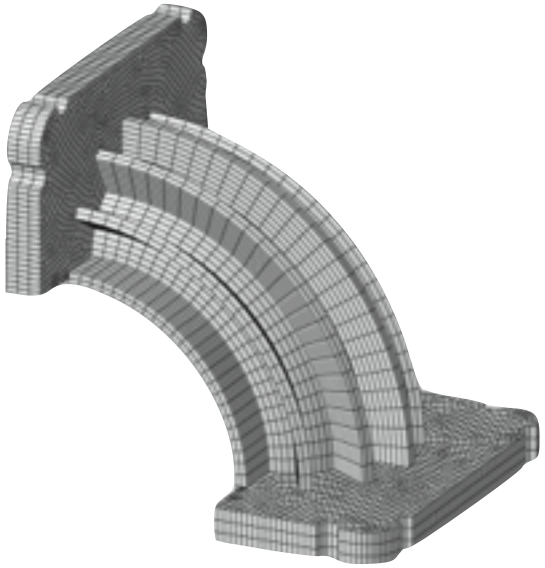


with E. Ruiz-Girones & J. Sarrate,
sliding nodes on curves and surfaces,
Best Technical Poster Award, IMR, 2014

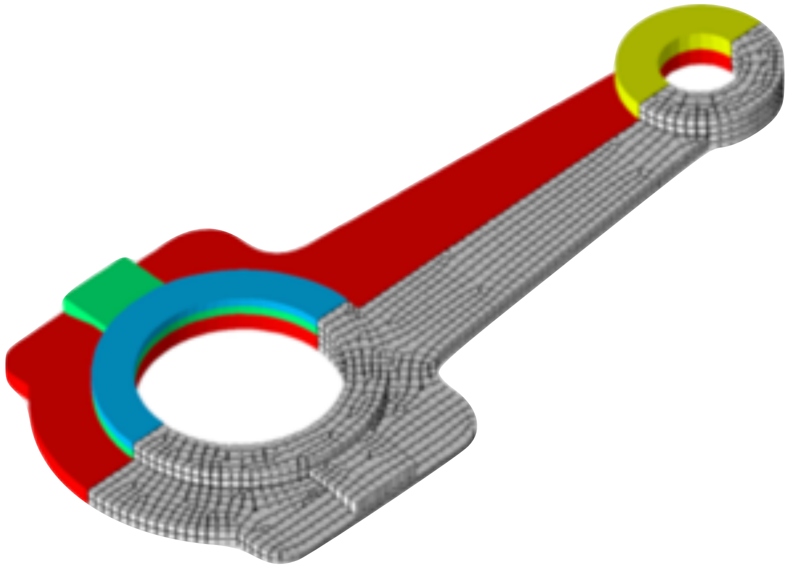
Non-structured hexahedral meshing

with E. Ruiz-Gironés, A. Huerta, J. Sarrate

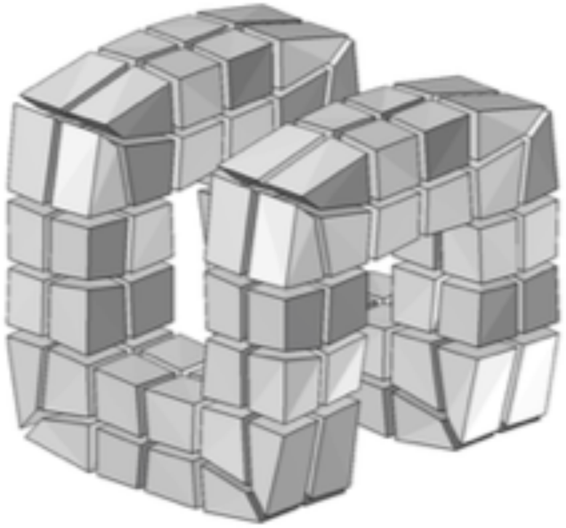
- **Problem**: approximate a domain with solution aligned elements
- **Solution**: non-structured hexahedral meshing
  - Met challenges: geometrical complexity, boundary layer alignment, …
  - Adequate: avoid locking, exploiting tensor product structure (SEM), composite materials …



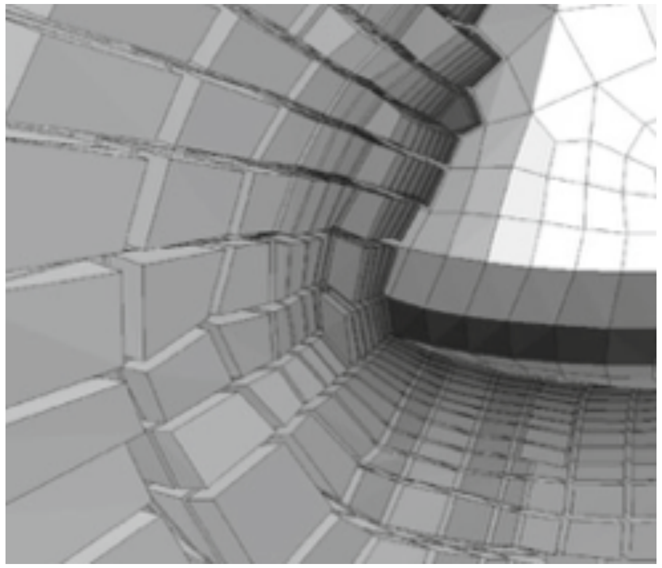
with J. Sarrate & A. Huerta, Sweeping, 2004, 2005



with E. Ruiz-Gironés & J. Sarrate, Multi-sweeping, 2008



with J. Sarrate, local dual contributions, 2009

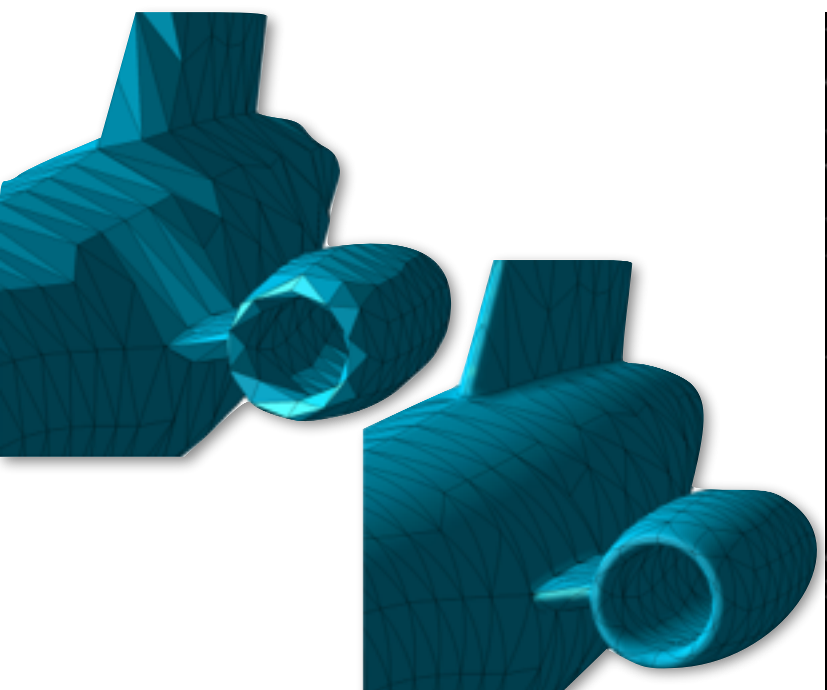


with E. Ruiz-Gironés & J. Sarrate, Receding front method, 2011

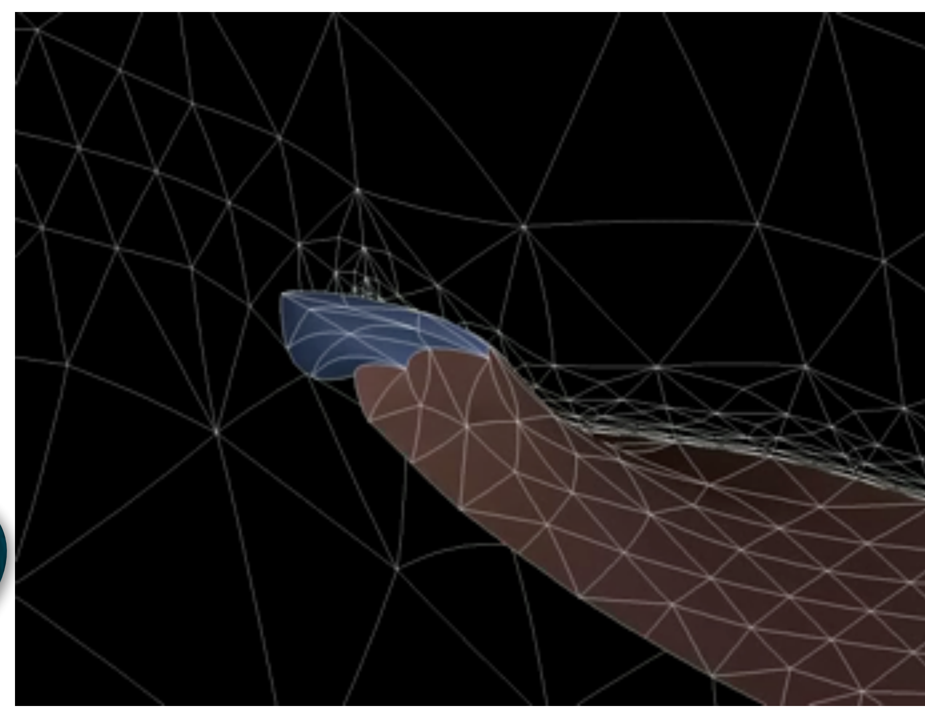
Curved meshing

with A. Gargallo, H. Chaurasia, E. Ruiz, P.-O. Persson, J. Peraire & J. Sarrate

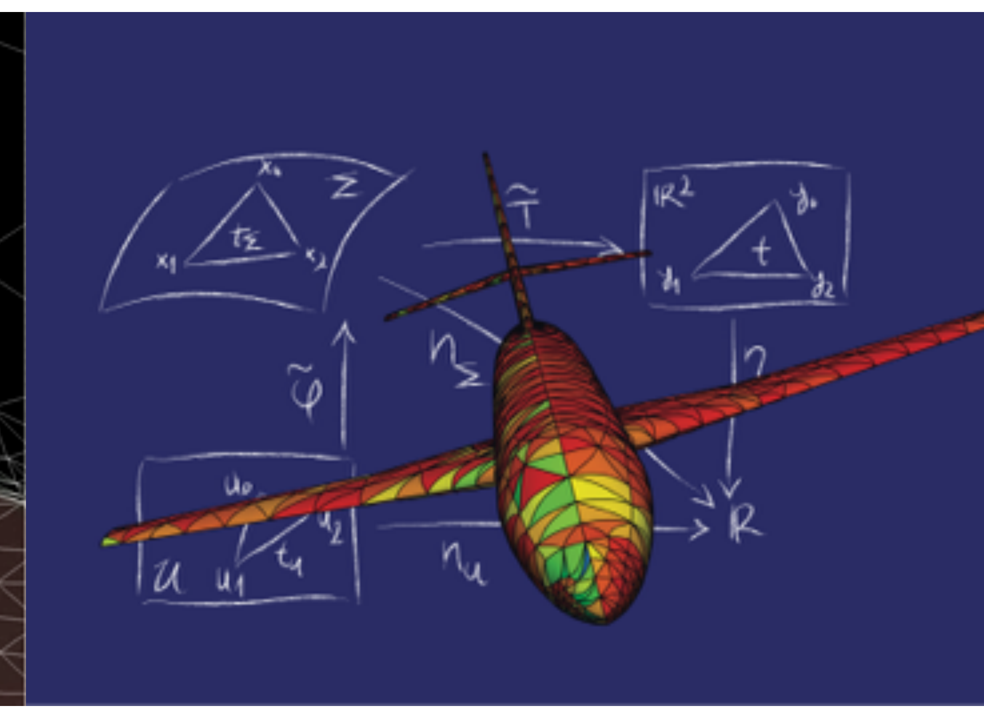
- **Problem**: accurate approximation of curved domains
- **Solution**: element-wise polynomial elements
  - Met challenges: accuracy, complexity, curvature, boundary layer, …
  - essential for high-order methods: convergence rates, enhance solver convergence …



straight-sided versus curved meshes, PhD dissertation, 2009



with H. Chaurasia, P.-O. Persson, J. Peraire, coarse-to-fine approach for moving meshes, Best Technical Poster Award, IMR, 2012

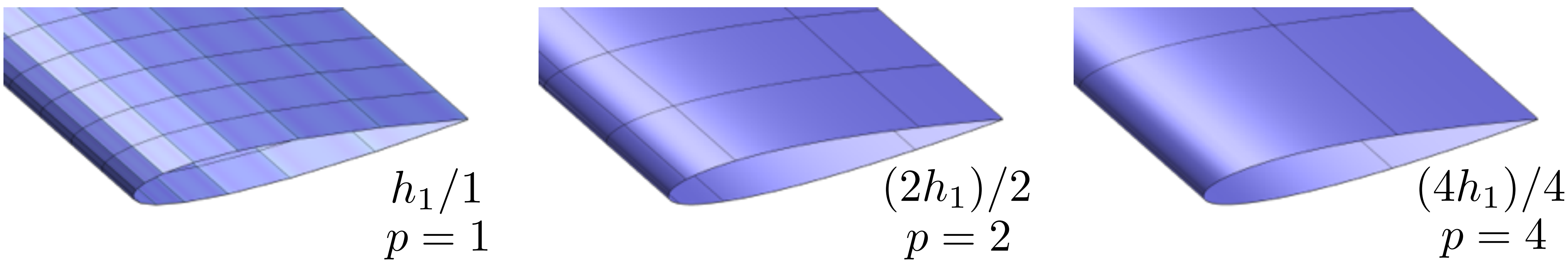


with A. Gargallo-Peiró, J. Peraire & J. Sarrate, curved meshing by quality optimization, Meshing Maestro Award, IMR, 2012

High-order: exponential convergence rate

- Convergence to a smooth geometry (smooth solution):
  - Approximate with element-wise polynomials (vs. linear)

    $\mathcal{O}\left(h^{p+1}\right)$ (vs. $\mathcal{O}\left(h^{2}\right)$) h : mesh size, p : polynomial degree

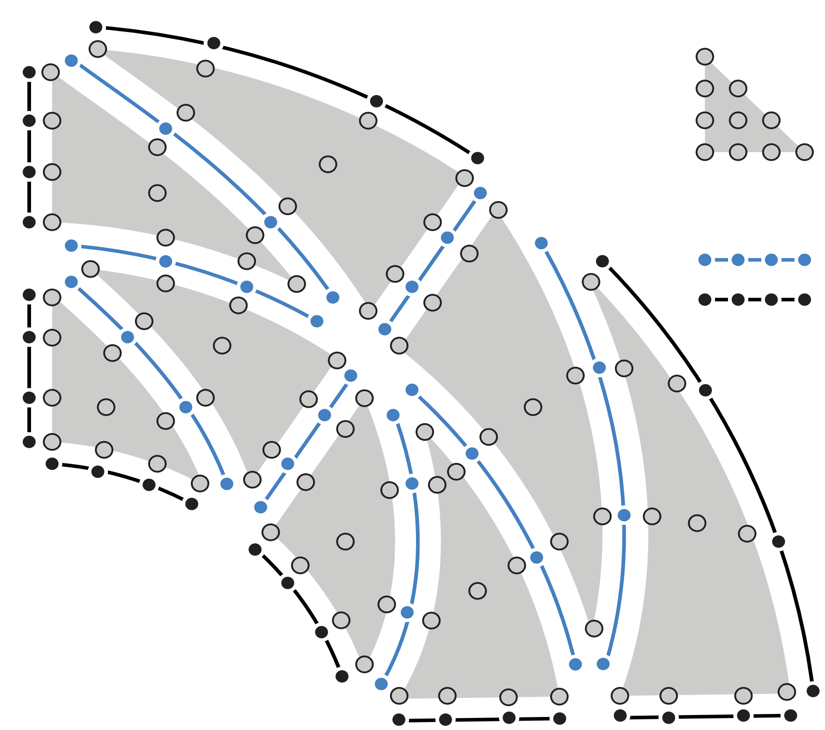


with E. Ruiz & J. Sarrate

- For same resolution (h/p) more accurate: potentially cheaper !
  - specially for high accuracies & implicit is required (complex flows)

    with Huerta, Angeloski & Peraire, IJNME'13
  - Hybridizable discontinuous Galerkin solver (GPU, distributed)

    with N.C. Nguyen, J. Peraire, AIAA'11,13



Summary & concluding remarks

- A “good mesh" accounts for: geometrical, physical & numerical fidelity, computational efficiency
- Basic concepts & packages for the meshing work flow:
  - Geometry representation
  - Preparing geometry for simulation (heal & defeature)
  - Element sizing
  - Element types
  - Meshing algorithms
- Approaches chosen according to: physics, geometry representation, geometry complexity, computational method, …
- Some research results on mesh generation
- A mesh is a main ingredient of a simulation

Books

- George, Paul-Louis and Houman Borouchaki, *Delaunay Triangulation and Meshing*, Hermes, Paris, 1998
- Thompson, Joe F, Bharat K. Soni, Nigel P. Weatherill, *Handbook of Grid Generation*, CRC Press, 1999
- Pascal Frey, Paul-Louis George, *Mesh Generation*, John Wiley & Sons, 2000

Thank you

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